

Accuracy, Decimal Places, Significant Figures & Standard Form

Focused chapter scope

Decimal places • significant figures • the IB three-significant-figure convention • choosing appropriate accuracy in context • zeros and significant digits • standard/scientific form $a \times 10^n$ • operations in standard form • estimation and reasonableness • upper and lower bounds • error intervals • percentage error • real-life reporting in finance, science, engineering, travel and data.

Design of this revised edition

This chapter is written as a guided lesson rather than a rounding rule-sheet. We begin with situations in which the *same calculator display* should be reported differently, then ask what information a rounded number is meant to preserve. Worked examples deliberately separate the question from the solution and slow down at the decision: *decimal places or significant figures? exact or decimal? which digit controls the rounding? what does the context actually require?*

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1 Accuracy is a decision about information

Imagine three people looking at numbers on the same morning.

A cafe manager sees a bill of EUR 19.876. A laboratory technician sees a mass of 0.0047286 g. A city planner sees a population estimate of 3 784 291. None of them should simply “keep three digits after the decimal point”. The useful digits depend on what the number means.

That is the central story of this chapter: **rounding is not deleting digits at random; it is choosing how much information to communicate.**

Two different languages of accuracy

Decimal places (d.p.) count positions *after the decimal point*. The location of the decimal point controls the rounding.

Significant figures (s.f.) count meaningful digits beginning at the first non-zero digit. The size of the number does not change how many significant digits are kept.

Worked Example 1 - three numbers, three reporting decisions

A cafe bill is EUR 19.876, a laboratory mass is 0.0047286 g, and a city population estimate is 3 784 291.

- Give the cafe bill to the nearest cent.
- Give the laboratory mass to 3 significant figures.
- Give the population to 3 significant figures and write it in standard form.

Solution

(a) Nearest cent means 2 decimal places. We keep 19.87 and inspect the next digit, 6. It rounds upward:

$$19.876 \longrightarrow \boxed{\text{EUR } 19.88}.$$

(b) For significant figures, leading zeros do not count. The first three significant digits are 4, 7, 2 and the next digit is 8, so

$$0.0047286 \longrightarrow \boxed{0.00473 \text{ g}}.$$

(c) The first three significant digits are 3, 7, 8 and the next digit is 4, so

$$3\,784\,291 \longrightarrow 3\,780\,000 = \boxed{3.78 \times 10^6}.$$

The same phrase “round the number” has produced three very different-looking answers because the contexts are different.

1.1 Decimal places: the decimal point is the anchor

When a question says “to n decimal places”, count exactly n positions to the right of the decimal point. The next digit decides whether the last kept digit stays or increases by 1.

Worked Example 2 - timing data

A runner’s split time is 87.3649 s. Give the time to

- 1 decimal place;
- 2 decimal places;
- 3 decimal places.

Solution

(a) To 1 d.p., keep 87.3 and inspect the next digit 6: 87.4 s .

(b) To 2 d.p., keep 87.36 and inspect the next digit 4: 87.36 s .

(c) To 3 d.p., keep 87.364 and inspect the next digit 9: 87.365 s .

Notice that the same raw measurement can legitimately be reported at several accuracies when the question specifies them.

1.2 Significant figures: the first non-zero digit is the anchor

The zeros before the first non-zero digit only position the decimal point. They are not significant. Once counting begins, zeros can become significant.

Which zeros count?

Number	Number of s.f.	Why
0.00472	3	leading zeros are placeholders
0.00520	3	the final decimal zero communicates precision
20.07	4	zeros between non-zero digits count
8.040	4	internal and trailing decimal zeros count
53 000	ambiguous alone	standard form states the intended precision clearly

Worked Example 3 - a small measurement

A sensor records 0.0063847 A. Give the reading to 3 significant figures.

Solution

Do *not* begin counting at the first zero. The significant digits are

$\underbrace{6}_{1\text{st}} \quad \underbrace{3}_{2\text{nd}} \quad \underbrace{8}_{3\text{rd}} \quad 4, 7.$

The next digit is 4, so the third significant digit stays unchanged:

0.00638 A .

Worked Example 4 - a large value

A transport model gives a distance of 84 763.2 m. Give the value to

- (a) 2 significant figures;
- (b) 3 significant figures;
- (c) 4 significant figures.

Solution

The first digits are 8, 4, 7, 6, 3, 2.

(a) Keep 84; the next digit is 7, so $84 \rightarrow 85$: $85\,000 \text{ m}$.

(b) Keep 847; the next digit is 6, so $847 \rightarrow 848$: $84\,800 \text{ m}$.

(c) Keep 8476; the next digit is 3, so no increase: $84\,760 \text{ m}$.

For large whole numbers, standard form is often the clearest way to show exactly how many significant figures were intended.

2 Decimal places and significant figures are not interchangeable

Students often know both definitions but still treat “3 d.p.” and “3 s.f.” as if they were nearly the same instruction. They are not. One counts from the decimal point; the other counts from the first non-zero digit.

Worked Example 5 - the same number rounded two different ways

For each value, give the result to 2 decimal places and to 3 significant figures.

- (i) 14.2768, (ii) 0.0063847, (iii) 98 643.2.

Solution

(i) 14.2768:

$$2 \text{ d.p. : } \boxed{14.28}, \quad 3 \text{ s.f. : } \boxed{14.3}.$$

(ii) 0.0063847:

$$2 \text{ d.p. : } \boxed{0.01}, \quad 3 \text{ s.f. : } \boxed{0.00638}.$$

This is the most important contrast: decimal-place rounding can be very coarse for a very small number.

(iii) 98 643.2:

$$2 \text{ d.p. : } \boxed{98\,643.20}, \quad 3 \text{ s.f. : } \boxed{98\,600}.$$

For a large number, 2 d.p. preserves almost everything, while 3 s.f. deliberately compresses the information.

Worked Example 6 - why a final zero can matter

A machine component is labelled 12.50 mm.

- (a) How many significant figures are shown?
 (b) Explain why 12.50 mm communicates something different from 12.5 mm.

Solution

(a) The digits 1, 2, 5, 0 are all significant, so the label contains $\boxed{4 \text{ s.f.}}$.

(b) The trailing zero tells us the value has been reported to the nearest 0.01 mm. By contrast, 12.5 mm normally communicates only the nearest 0.1 mm. The numerical size is the same, but the *stated precision* is different.

Worked Example 7 - negative numbers

A weather station records -27.9581°C . Give the temperature to 3 significant figures.

Solution

The minus sign is not a significant digit. Round the magnitude 27.9581: keep 2, 7, 9 and inspect the next digit 5. Thus 27.9 rounds to 28.0.

$$\boxed{-28.0^\circ\text{C}}.$$

The zero is essential: -28.0 has three significant figures, while -28 has only two.

The IB accuracy rule - make this automatic

In IB Mathematics AA, unless a question states a different accuracy, numerical answers should be given **exactly or correct to three significant figures**. In practice:

- keep exact forms such as $\sqrt{73}$, $\frac{17}{6}$ or $\frac{\pi}{3}$ when exactness is appropriate;
- if you give a decimal and no other accuracy is specified, use **3 significant figures**;
- if the question says “nearest degree”, “2 d.p.”, “nearest cent”, and so on, that instruction overrides the general rule;
- do not round intermediate calculator values unnecessarily.

Worked Example 8 - exact or 3 significant figures?

Evaluate $\sqrt{73}$ and state an answer suitable for an IB examination.

Solution

The exact answer is

$$\boxed{\sqrt{73}}.$$

If a decimal answer is required, the calculator gives

$$\sqrt{73} = 8.5440037\dots,$$

so to 3 significant figures,

$$\boxed{8.54}.$$

Writing 8.544 would normally be an unnecessary 4-significant-figure decimal unless the question requests it.

Worked Example 9 - premature rounding can change the final answer

Calculate

$$\frac{2.37}{1.28} \times 5.27$$

and give the final answer to 3 significant figures. Then show what happens if the intermediate quotient is rounded too early.

Solution

Using the full calculator value,

$$\frac{2.37}{1.28} \times 5.27 = 9.757734375\dots,$$

so the correct final result is

$$\boxed{9.76}.$$

If we prematurely replace $2.37/1.28 = 1.8515625$ by 1.85, then

$$1.85(5.27) = 9.7495 \rightarrow 9.75 \quad (3 \text{ s.f.}),$$

which is different. The safe habit is simple: **store full values, round once at the end.**

3 Standard form: a zoom lens for very large and very small numbers

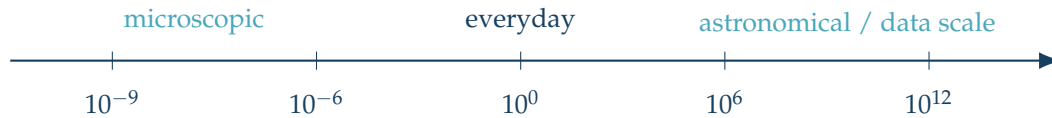
When numbers become extremely large or extremely small, ordinary decimal notation hides the scale. Standard form separates two pieces of information: the significant digits and the power of ten.

Standard (scientific) form

A non-zero number is written in standard form as

$$\boxed{a \times 10^n}, \quad 1 \leq |a| < 10, \quad n \in \mathbb{Z}.$$

The coefficient a carries the significant digits. The exponent n tells us the scale.

**Worked Example 10 - from an ordinary large number to standard form**

Write 384 400 000 in standard form, first preserving all shown significant digits and then to 3 significant figures.

Solution

Move the decimal point until the leading number lies between 1 and 10:

$$384\,400\,000 = 3.844 \times 10^8.$$

This already has four significant figures. To 3 significant figures,

$$3.844 \times 10^8 \longrightarrow \boxed{3.84 \times 10^8}.$$

Worked Example 11 - a microscopic scale

Write 0.00000007246 m in standard form and then give it to 3 significant figures.

Solution

The decimal point moves eight places to the right to create 7.246, so

$$0.00000007246 = 7.246 \times 10^{-8} \text{ m}.$$

To 3 significant figures,

$$\boxed{7.25 \times 10^{-8} \text{ m}}.$$

The negative exponent records the very small scale; it does not mean the number itself is negative.

Worked Example 12 - multiplication in standard form

A data system processes 3.2×10^6 records, each requiring 4.5×10^3 operations. Find the total number of operations in standard form.

Solution

Multiply the coefficients and add the powers:

$$(3.2 \times 10^6)(4.5 \times 10^3) = 14.4 \times 10^9.$$

The coefficient 14.4 is not yet in standard form, so shift one place:

$$14.4 \times 10^9 = \boxed{1.44 \times 10^{10}}.$$

Worked Example 13 - addition requires a common power

Compute

$$4.7 \times 10^6 + 8.2 \times 10^5$$

and write the result in standard form.

Solution

The powers must match before the coefficients can be added:

$$8.2 \times 10^5 = 0.82 \times 10^6.$$

Therefore

$$4.7 \times 10^6 + 0.82 \times 10^6 = \boxed{5.52 \times 10^6}.$$

A common mistake is to add $4.7 + 8.2$ while ignoring that the powers of ten are different.

Worked Example 14 - real data storage

A cloud archive contains 2.85×10^9 images, each using an average of 3.6×10^6 bytes. Estimate the total storage and then calculate it exactly from the given values, giving the final answer to 3 significant figures.

Solution

A quick estimate uses 3×10^9 and 4×10^6 :

$$(3 \times 10^9)(4 \times 10^6) \approx 1.2 \times 10^{16} \text{ bytes.}$$

Now calculate using the given values:

$$(2.85 \times 10^9)(3.6 \times 10^6) = 10.26 \times 10^{15} = 1.026 \times 10^{16}.$$

To 3 significant figures,

$$\boxed{1.03 \times 10^{16} \text{ bytes}}.$$

The estimate tells us immediately that the exponent 10^{16} is sensible.

4 Estimation: the calculator's reality check

An estimate is not a poor version of the calculation. It is a different tool. Before trusting a screen full of digits, ask roughly what size the answer should have.

Worked Example 15 - fuel efficiency

A car travels 643 km using 52.7 L of fuel.

- Estimate the fuel efficiency in km/L.
- Calculate the value and give the final answer in IB form.

Solution

(a) A useful mental estimate is

$$\frac{640}{50} \approx 12.8 \text{ km/L.}$$

(b) Using the unrounded values,

$$\frac{643}{52.7} = 12.2011 \dots,$$

so the IB-style numerical answer is

$$\boxed{12.2 \text{ km/L}} \quad (3 \text{ s.f.}).$$

The accurate result is close to the estimate, so there is no obvious scale error.

Worked Example 16 - spotting an impossible calculator entry

A student enters a calculation for $42.1/5170$ and writes 0.0814. Explain, without repeating the exact calculation, why the answer cannot be right.

Solution

Round mentally:

$$\frac{42.1}{5170} \approx \frac{40}{5000} = 0.008.$$

So the true answer should be around eight thousandths, not eight hundredths. The student's answer is about ten times too large, most likely because of a misplaced decimal point.

5 Bounds: what a rounded measurement is hiding

A rounded value is not a single exact measurement. It represents an interval of possible original values.

If a length is reported as 1.8 m to the nearest 0.1 m, values from 1.75 m up to (but not including) 1.85 m would all round to 1.8 m.

**The half-unit rule**

If a value is rounded to the nearest unit u , then the half-unit is $u/2$.

$$\text{lower bound} = \text{stated value} - \frac{u}{2}, \quad \text{upper bound} = \text{stated value} + \frac{u}{2}.$$

The standard error interval is written

$$\boxed{\text{lower bound} \leq x < \text{upper bound}}.$$

The upper endpoint is excluded because it would round to the next stated value.

Worked Example 17 - a measurement to the nearest tenth

A table length is recorded as 1.8 m to the nearest 0.1 m. Write the error interval.

Solution

The rounding unit is 0.1 m, so the half-unit is 0.05 m:

$$1.8 - 0.05 = 1.75, \quad 1.8 + 0.05 = 1.85.$$

Hence

$$1.75 \leq L < 1.85 \text{ m}.$$

Worked Example 18 - bounds from significant figures

A road distance is reported as 73.5 km correct to 3 significant figures. Find its lower and upper bounds.

Solution

At 73.5, the third significant figure is the tenths digit. Therefore the value was rounded to the nearest 0.1 km, giving half-unit 0.05 km:

$$73.45 \leq d < 73.55 \text{ km}.$$

The phrase “3 significant figures” must first be translated into the actual place value being rounded.

Worked Example 19 - bounds for area

A rectangular display is measured as 42.6 cm by 24.0 cm, with each dimension correct to the nearest 0.1 cm. Find the lower bound and upper-bound value for its area.

Solution

The dimensions satisfy

$$42.55 \leq L < 42.65, \quad 23.95 \leq W < 24.05.$$

Because all quantities are positive, the smallest area uses both lower bounds:

$$A_{\min} = 42.55(23.95) = 1019.0725 \text{ cm}^2.$$

The upper-bound value uses both upper bounds:

$$A_{\max} = 42.65(24.05) = 1025.7325 \text{ cm}^2.$$

So

$$1019.0725 \leq A < 1025.7325 \text{ cm}^2.$$

If a final numerical answer is requested without another accuracy instruction, it would normally be reported to 3 significant figures.

Worked Example 20 - bounds for a quotient

A journey distance is 128 km to the nearest kilometre and the travel time is 1.70 h to the nearest 0.01 h. Find lower and upper bounds for the mean speed.

Solution

The measurements satisfy

$$127.5 \leq d < 128.5, \quad 1.695 \leq t < 1.705.$$

For positive quantities, the smallest quotient uses the smallest numerator and largest denominator:

$$v_{\min} = \frac{127.5}{1.705} = 74.7800 \dots$$

The largest quotient uses the largest numerator and smallest denominator:

$$v_{\max} = \frac{128.5}{1.695} = 75.8112 \dots$$

Therefore

$$74.7800 \dots \leq v < 75.8112 \dots \text{ km/h}.$$

As a practical summary, the speed is between approximately 74.8 and 75.8 km/h.

6 Percentage error and sensible reporting

Sometimes we compare an experimental or rounded value with an accepted value. A common measure is

$$\text{percentage error} = \frac{|\text{measured} - \text{accepted}|}{|\text{accepted}|} \times 100\%.$$

Worked Example 21 - comparing an experiment with an accepted value

A laboratory experiment gives 4.72 s while an accepted value is 4.68 s. Find the percentage error, giving the final answer to 3 significant figures.

Solution

The absolute error is

$$|4.72 - 4.68| = 0.04 \text{ s}.$$

Therefore

$$\frac{0.04}{4.68} \times 100 = 0.854700 \dots \%$$

To 3 significant figures,

$$0.855\%.$$

Notice that 0.855% has three significant figures even though it also happens to have three decimal places. Those are separate facts.

Worked Example 22 - an engineering label in standard form

A thin film is reported as 7.20×10^{-5} m correct to 3 significant figures.

- Write the value as an ordinary decimal.
- State its error interval in standard form.

Solution

(a)

$$7.20 \times 10^{-5} = 0.0000720 \text{ m}.$$

The final zero is significant.

(b) The third significant figure is in the 10^{-7} place, so the half-unit is 5×10^{-8} :

$$7.20 \times 10^{-5} - 5 \times 10^{-8} = 7.195 \times 10^{-5},$$

$$7.20 \times 10^{-5} + 5 \times 10^{-8} = 7.205 \times 10^{-5}.$$

Hence

$$7.195 \times 10^{-5} \leq x < 7.205 \times 10^{-5}.$$

7 A final decision strategy

In an exam, accuracy questions become much easier if you ask the decisions in the same order every time.

CF Maths accuracy checklist

1. **What does the question explicitly ask for?** Nearest cent, d.p., s.f., exact value, standard form, bounds?
2. **If nothing is stated, what is the IB convention?** Exact or 3 significant figures.
3. **Where does counting begin?** Decimal point for d.p.; first non-zero digit for s.f.
4. **Which next digit controls the rounding?** 0–4: stay; 5–9: increase.
5. **Have you kept guard digits?** Avoid premature rounding in intermediate steps.
6. **Does the result make sense in context?** Estimate the scale and check units.
7. **If the input was rounded, are bounds required?** Translate the final stated digit into a rounding unit, then use half of it.

Worked Example 23 - full IB-style synthesis

A laboratory sample has mass 48.6 g correct to the nearest 0.1 g and volume 17.2 cm³ correct to the nearest 0.1 cm³.

- (a) Calculate the density using the stated measurements and give the answer in IB form.
- (b) Find lower and upper bounds for the density.
- (c) Explain why part (b) gives more information than simply writing more decimal places in part (a).

Solution

(a)

$$\rho = \frac{48.6}{17.2} = 2.825581 \dots,$$

so

$$\boxed{\rho = 2.83 \text{ g cm}^{-3}} \quad (3 \text{ s.f.}).$$

(b)

$$48.55 \leq m < 48.65, \quad 17.15 \leq V < 17.25.$$

For positive values,

$$\rho_{\min} = \frac{48.55}{17.25} = 2.81449 \dots,$$

$$\rho_{\max} = \frac{48.65}{17.15} = 2.83673 \dots$$

Hence

$$\boxed{2.81449 \dots \leq \rho < 2.83673 \dots \text{ g cm}^{-3}}.$$

- (c) More decimal places in the nominal answer do not create new measurement information. The bounds show the range of densities that are genuinely consistent with the rounded measurements.

Final IB reminder: exact if appropriate; otherwise 3 significant figures, unless the question tells you differently.