

CF MATHS
IB Mathematics AA

Accuracy, Significant Figures & Standard Form Worked Solutions

Full working • Exact values where appropriate • IB-style final accuracy

These solutions are deliberately spacious. The important step is often not the arithmetic itself but the decision about *which digit is controlling the accuracy, whether the result should remain exact, and when rounding should occur.*

Question 1 - Decimal places versus significant figures

(a) 14.2768

$$2 \text{ d.p.} : 14.2768 \rightarrow \boxed{14.28}, \quad 3 \text{ s.f.} : 14.2768 \rightarrow \boxed{14.3}.$$

(b) 0.0063847

$$2 \text{ d.p.} : \boxed{0.01}, \quad 3 \text{ s.f.} : \boxed{0.00638}.$$

The leading zeros do not count as significant figures.

(c) 98 643.2

$$2 \text{ d.p.} : \boxed{98\,643.20}, \quad 3 \text{ s.f.} : \boxed{98\,600}.$$

This question shows why d.p. and s.f. cannot be treated as interchangeable instructions.

Question 2 - Zeros and significant figures

(a) 0.00520 has $\boxed{3}$ significant figures: 5, 2, 0. The leading zeros only locate the decimal point; the final zero communicates precision.

(b) 20.07 has $\boxed{4}$ significant figures. The zeros lie between non-zero digits and therefore count.

(c) 9.000 has $\boxed{4}$ significant figures. The three trailing decimal zeros are part of the stated precision.

(d) 0.0009040 has $\boxed{4}$ significant figures: 9, 0, 4, 0.

Question 3 - Context decides the reporting rule

(a) Nearest cent means 2 decimal places:

$$47.836 \rightarrow \boxed{\text{EUR } 47.84}.$$

This is a decimal-place instruction.

(b) Nearest 0.01 s is again 2 decimal places:

$$12.4862 \rightarrow \boxed{12.49 \text{ s}}.$$

(c)

$$6\,284\,913 \rightarrow 6\,280\,000 = \boxed{6.28 \times 10^6}.$$

This is a significant-figure instruction.

(d)

$$0.0047286 \rightarrow \boxed{0.00473 \text{ mm}} \quad (3 \text{ s.f.}).$$

Again, significant figures are counted from the first non-zero digit.

Question 4 - Exact or three significant figures

(a) The exact answer is

$$\boxed{\sqrt{73}}.$$

(b)

$$\sqrt{73} = 8.5440037 \dots \rightarrow \boxed{8.54} \quad (3 \text{ s.f.}).$$

(c) 8.544 is a four-significant-figure decimal. Under the normal IB convention, if no other accuracy is requested, a numerical decimal answer should be correct to three significant figures; alternatively, the exact form $\sqrt{73}$ may be kept.

Question 5 - Premature rounding

(a) Keeping the full calculator value,

$$\frac{2.37}{1.28} \times 5.27 = 9.757734375 \dots$$

so

$$\boxed{9.76} \quad (3 \text{ s.f.}).$$

(b) If $2.37/1.28$ is prematurely rounded to 1.85, then

$$1.85(5.27) = 9.7495 \rightarrow \boxed{9.75} \quad (3 \text{ s.f.}).$$

(c) The two final answers differ. Keep full calculator precision through intermediate steps and round the final requested value only.

Question 6 - Converting standard form

(a)

$$72\,400\,000 = \boxed{7.24 \times 10^7}.$$

(b)

$$0.0000385 = \boxed{3.85 \times 10^{-5}}.$$

(c)

$$6.02 \times 10^7 = \boxed{60\,200\,000}.$$

(d)

$$4.70 \times 10^{-6} = \boxed{0.00000470}.$$

The final zero is retained because 4.70 contains three significant figures.

Question 7 - Operations in standard form

(a)

$$(2.5 \times 10^6)(3.2 \times 10^4) = 8.0 \times 10^{10} = \boxed{8 \times 10^{10}}.$$

(b)

$$\frac{9.6 \times 10^{-3}}{3.2 \times 10^5} = 3 \times 10^{-8} = \boxed{3 \times 10^{-8}}.$$

(c) First match powers:

$$5.3 \times 10^8 + 7.4 \times 10^7 = 5.3 \times 10^8 + 0.74 \times 10^8 = \boxed{6.04 \times 10^8}.$$

(d)

$$8.1 \times 10^{-5} - 2.6 \times 10^{-5} = \boxed{5.5 \times 10^{-5}}.$$

Question 8 - Cloud storage

(a) Using one significant figure,

$$(3 \times 10^9)(4 \times 10^6) = \boxed{1.2 \times 10^{16}} \text{ bytes (estimate).}$$

(b)

$$(2.85 \times 10^9)(3.6 \times 10^6) = 10.26 \times 10^{15} = 1.026 \times 10^{16} \text{ bytes.}$$

(c) To three significant figures,

$$\boxed{1.03 \times 10^{16} \text{ bytes}}.$$

The accurate value has the same order of scale as the estimate.

Question 9 - Comparing scales

The second wavelength is larger because 10^{-5} is one hundred times the scale of 10^{-7} before coefficients are considered.

(a) Larger wavelength:

$$5.4 \times 10^{-5} \text{ m}.$$

(b)

$$\frac{5.4 \times 10^{-5}}{7.2 \times 10^{-7}} = \frac{5.4}{7.2} \times 10^2 = 0.75 \times 10^2 = 75.0.$$

Thus it is 75.0 times larger to three significant figures.

(c) The exponent difference is 2, so we expect a ratio on the scale of 10^2 ; the coefficient ratio then adjusts that estimate.

Question 10 - Fuel efficiency and estimation

(a) A sensible estimate is

$$\frac{640}{50} \approx 12.8 \text{ km/L}.$$

Any nearby convenient estimate around 12–13 km/L is reasonable.

(b)

$$\frac{643}{52.7} = 12.2011 \dots \rightarrow 12.2 \text{ km/L} \quad (3 \text{ s.f.}).$$

(c) 122 km/L is roughly ten times the estimate, so it is not plausible. A decimal-place or data-entry error is likely.

Question 11 - Basic error intervals

(a) Nearest 0.1 gives half-unit 0.05:

$$1.75 \leq L < 1.85 \text{ m}.$$

(b) 2 decimal places means nearest 0.01 kg, so half-unit 0.005 kg:

$$4.255 \leq m < 4.265 \text{ kg}.$$

(c) Nearest kilometre gives half-unit 0.5 km:

$$127.5 \leq d < 128.5 \text{ km}.$$

Question 12 - Bounds from significant figures

(a) In 73.5, the third significant figure is the tenths digit, so the rounding unit is

$$0.1 \text{ km}.$$

(b) Half-unit is 0.05 km:

$$73.45 \leq d < 73.55 \text{ km}.$$

(c) The absolute place value of the third significant digit depends on the size of the number. For example, three significant figures might mean nearest 0.001 for a small decimal or nearest 100 for a large whole number.

Question 13 - Area bounds

(a)

$$42.55 \leq L < 42.65, \quad 23.95 \leq W < 24.05.$$

(b) For positive dimensions,

$$A_{\min} = 42.55(23.95) = 1019.0725 \text{ cm}^2.$$

(c)

$$A_{\max} = 42.65(24.05) = 1025.7325 \text{ cm}^2.$$

Hence $1019.0725 \leq A < 1025.7325$.

(d) The nominal area is

$$42.6(24.0) = 1022.4 \rightarrow 1.02 \times 10^3 \text{ cm}^2 \quad (3 \text{ s.f.}).$$

This is a central reported value, not the full range of possible areas compatible with the rounded measurements.

Question 14 - Speed bounds

(a)

$$127.5 \leq d < 128.5, \quad 1.695 \leq t < 1.705.$$

(b) Minimum speed uses the smallest distance and largest time:

$$v_{\min} = \frac{127.5}{1.705} = 74.780058 \dots$$

(c) Maximum speed uses the largest distance and smallest time:

$$v_{\max} = \frac{128.5}{1.695} = 75.811209 \dots$$

(d) To three significant figures,

$$74.8 \leq v < 75.8 \text{ km/h}.$$

The unrounded bound values should be used during any further calculation.

Question 15 - Percentage error

(a)

$$|4.72 - 4.68| = \boxed{0.04 \text{ s}}.$$

(b)

$$\frac{0.04}{4.68} \times 100 = 0.854700 \dots \% \rightarrow \boxed{0.855\%}.$$

(c) Significant figures count from the first non-zero digit: 8, 5, 5. Decimal places count digits after the decimal point. Here the answer happens to satisfy both descriptions, but the concepts remain different.

Question 16 - Engineering precision

(a) 8.40 contains $\boxed{3 \text{ significant figures}}$.

(b) Nearest 0.01 mm gives half-unit 0.005 mm:

$$\boxed{8.395 \leq d < 8.405 \text{ mm}}.$$

(c) Since $r = d/2$,

$$\boxed{4.1975 \leq r < 4.2025 \text{ mm}}.$$

(d) Writing 8.4 would normally communicate only two significant figures and a resolution of 0.1 mm, so the hundredths-place information would be lost.

Question 17 - Density from rounded measurements

(a)

$$\rho = \frac{48.6}{17.2} = 2.825581 \dots \rightarrow \boxed{2.83 \text{ g cm}^{-3}}.$$

(b) The mass and volume intervals are

$$48.55 \leq m < 48.65, \quad 17.15 \leq V < 17.25.$$

Minimum density uses minimum mass and maximum volume:

$$\rho_{\min} = \frac{48.55}{17.25} = \boxed{2.814492 \dots \text{ g cm}^{-3}}.$$

(c) Maximum density uses maximum mass and minimum volume:

$$\rho_{\max} = \frac{48.65}{17.15} = \boxed{2.836734 \dots \text{ g cm}^{-3}}.$$

So the physically consistent interval is

$$2.814492 \dots \leq \rho < 2.836734 \dots$$

(d) Extra calculator digits in 2.8255814 do not recover information that was never present in the rounded mass and volume. The bounds describe the measurement uncertainty more honestly.

Question 18 - Satellite sensor synthesis

The recorded area is

$$8.6472391 \times 10^4 = 86\,472.391 \text{ m}^2.$$

(a) To two decimal places,

$$\boxed{86\,472.39 \text{ m}^2}.$$

(b) To three significant figures,

$$8.6472391 \times 10^4 \rightarrow \boxed{8.65 \times 10^4 \text{ m}^2}.$$

(c) Number of readings:

$$N = \frac{2.40 \times 10^4}{0.0376} = 638\,297.872 \dots = 6.38297872 \times 10^5.$$

Thus

$$\boxed{6.38 \times 10^5 \text{ readings}} \quad (3 \text{ s.f.}).$$

(d) One-significant-figure estimate:

$$\frac{2 \times 10^4}{0.04} = 5 \times 10^5,$$

so an accurate answer of order 10^5 is reasonable.(e) Nearest 0.01 m^2 gives half-unit 0.005 m^2 :

$$\boxed{86\,472.385 \leq A < 86\,472.395 \text{ m}^2}.$$