

CF MATHS

IB Mathematics AA Lecture Notes

Financial Applications

Compound Interest • TVM Solver • Loans • Annuities • Inflation • Geometric Series

How these notes are written

Financial mathematics is not a collection of calculator buttons. It is a story about money moving through time. In every example we first decide what is happening to the money, then translate that story into a timeline, then decide the signs, and only then use a formula or the financial solver. You will repeatedly see phrases such as “now we are going to identify the payment period”, “now we are going to evaluate the future value”, and “before pressing solve, let us check the signs”. That is deliberate: the aim is to teach the decision-making, not only the arithmetic.

Focused chapter scope

Percentage growth and depreciation; the connection with geometric sequences and series; compound interest; the TVM variables N , $I\%$, PV , PMT , FV , P/Y , C/Y ; two equivalent encodings for lump-sum investments; regular deposits and withdrawals; loans and amortization; inflation and real value; changing rates; calculator strategy; past-paper style Section B synthesis.

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1 The story behind compound interest

1.1 One multiplier, repeated again and again

Imagine that you place EUR 1000 in an account paying 5% interest each year. At the end of the first year the bank does not add a fixed number forever; it adds 5% of the amount currently in the account. So after one year,

$$1000(1.05).$$

Now we are going to move to the second year. The important detail is that the second 5% is applied to the *new* balance, not to the original EUR 1000. Therefore after two years,

$$1000(1.05)^2.$$

After n years the same multiplier has been applied n times.

Compound growth as a geometric sequence

If the effective rate per period is i (written as a decimal), then

$$FV = PV(1 + i)^n.$$

The balances $PV, PV(1 + i), PV(1 + i)^2, \dots$ form a geometric sequence with common ratio $1 + i$.

Key insight. A percentage change creates a multiplier. An increase of $r\%$ means multiply by $1 + r/100$; a decrease of $r\%$ means multiply by $1 - r/100$. If the same multiplier repeats, the model is geometric.

Worked Example 1 - investment growth as a geometric sequence

An investment of EUR 8500 earns 4.2% per year, compounded annually.

(a) State the common ratio of the year-end balances. (b) Find the value after 12 years. (c) Explain why the model is geometric rather than arithmetic.

Solution. Let us start with the story rather than the formula. Every year the account keeps all of its previous value and gains an extra 4.2%. So each year we multiply by

$$1 + \frac{4.2}{100} = 1.042.$$

That number is the common ratio. Now we are going to apply the same multiplier for 12 years:

$$FV = 8500(1.042)^{12} = 13926.1655 \dots$$

so the value is approximately EUR 13926.17.

Why geometric? Because the *percentage* increase is constant. The euro increase is not constant: as the balance becomes larger, 4.2% of the balance also becomes larger. That rules out an arithmetic model.

Answer: common ratio 1.042; EUR 13926.17

1.2 Depreciation is the same mathematics with a multiplier below 1

Suppose a car loses 17% of its value every year. It keeps 83%, so its yearly multiplier is 0.83. The model is still geometric; the only difference is that the common ratio lies between 0 and 1.

$$V_n = V_0(0.83)^n.$$

Trap. Do not subtract 17% once and then keep subtracting that same euro amount. Compound depreciation means 17% of the *current* value is lost each year.

Worked Example 2 - annual depreciation and a replacement threshold

A machine costs EUR 48000 and loses 18% of its value each year.

(a) Find its value after 7 years. (b) The company replaces it when its value first falls below EUR 12000. Determine when this occurs.

Solution. First we translate “loses 18%” into the multiplier 0.82. Now we are going to apply that multiplier seven times:

$$V_7 = 48000(0.82)^7 = 11965.7026 \dots$$

so after 7 years the value is about EUR 11965.70.

For the threshold, we are not asking for an exact continuous time; the machine is checked after complete years. We solve

$$48000(0.82)^n < 12000.$$

A logarithm or GDC gives $n > 6.99 \dots$ Now we interpret the decimal. The first *complete* year satisfying the condition is year 7.

IB exam habit. Whenever the question says “first complete year/month”, solve continuously first, then test or round in the direction required by the inequality.

Answer: EUR 11965.70; after 7 complete years

2 Meet the financial solver

2.1 The timeline comes before the buttons

Before entering anything into a GDC, imagine a horizontal time axis. At time 0 there may be an initial deposit or a loan. During the timeline there may be regular payments. At the end there is a final balance or target. The TVM solver simply gives names to these cash flows.

The seven TVM fields

N	Number of <i>payment periods</i> . Monthly payments for 5 years give $N = 60$; annual payments for 5 years give $N = 5$.
$I\%$	Nominal annual interest rate entered as a percentage. For 6%, enter 6, not 0.06.
PV	Present value at time 0. It is a cash flow occurring now.
PMT	Regular payment made every payment period.
FV	Future value after the final payment period.
P/Y	Payments per year. Monthly $\rightarrow 12$, quarterly $\rightarrow 4$, annual $\rightarrow 1$.
C/Y	Compounding periods per year. Monthly $\rightarrow 12$, quarterly $\rightarrow 4$, half-yearly $\rightarrow 2$, annual $\rightarrow 1$.

2.2 Why some values are positive and others negative

This is where many students lose marks even though they understand the mathematics. The calculator needs to know the *direction* of each cash flow. The simplest rule is to stand inside the story and ask: *is the money entering my pocket or leaving my pocket?*

If you invest EUR 10000 today, that money leaves your pocket now, so from your point of view $PV = -10000$. If you later withdraw EUR 15000, that future money enters your pocket, so $FV = +15000$. The signs are opposite because the cash-flow directions are opposite.

For a loan the story reverses. The bank gives you EUR 200000 today, so the money enters your pocket and $PV = +200000$. Your monthly repayments leave your pocket, so $PMT < 0$. If the loan is completely repaid, there is no balance left at the end, so $FV = 0$.

Key insight. The calculator does not know what “saving” or “borrowing” means. It only knows positive and negative cash flows. Opposite directions must have opposite signs.

Trap. If PV , PMT and FV all have the same sign, stop. In almost every standard IB finance story, that means the cash-flow directions have been entered incorrectly.

Worked Example 3 - reading the story into the TVM fields

Sara invests EUR 10000 for 6 years at 5% per year, compounded annually, with no additional deposits. Write a complete TVM setup to find the final value.

Solution. Now we are going to translate one sentence at a time.

There are 6 annual periods, so $N = 6$. The interest rate is 5%, so $I\% = 5$. Sara pays EUR 10000 into the account now; from her point of view it leaves her pocket, so $PV = -10000$. There are no repeated contributions, so $PMT = 0$. Both payment and compounding periods are yearly, so $P/Y = C/Y = 1$. The unknown is the amount she can receive later, so $FV = ?$ and it will come out positive.

$$N = 6, \quad I\% = 5, \quad PV = -10000, \quad PMT = 0, \quad FV = ?, \quad P/Y = 1, \quad C/Y = 1.$$

Before pressing solve, notice the sign story: money out now, money back later. That is exactly why PV and FV have opposite signs.

2.3 GDC route: Casio fx-CG50 and TI-Nspire

Casio fx-CG50. MENU → Financial → Compound Interest (CMPD). Enter the known values. In SET UP use Payment = End unless the question explicitly says deposits occur at the beginning of each period. Move to the unknown field and solve.

TI-Nspire CX/CX II. Calculator → Menu → Finance → Finance Solver. Enter the known values, including PpY , CpY and $PmtAt$. Move to the unknown and press Enter.

IB exam habit. Write the setup on paper before using the calculator. In an exam, a correct final number with no visible financial setup may hide a wrong method. Writing N , $I\%$, PV , PMT , FV , P/Y and C/Y makes the reasoning visible.

3 Lump-sum investments: solving for any unknown

3.1 Two equivalent encodings when $PMT = 0$

This is one of the most useful calculator ideas in the chapter. Suppose money is invested for t years, compounded k times per year, and there are no regular payments.

There are two perfectly valid ways to describe the same timeline.

Two encodings for a lump sum

Method A: make the payment period equal the compounding period.

$$P/Y = C/Y = k, \quad N = kt.$$

For 4 years of monthly compounding: $P/Y = C/Y = 12$ and $N = 48$.

Method B: keep one payment period per year.

$$P/Y = 1, \quad C/Y = k, \quad N = t.$$

For the same investment: $P/Y = 1$, $C/Y = 12$ and $N = 48$.

These are equivalent only because $PMT = 0$. There is no repeated payment whose frequency needs to be preserved.

Trap. Do not use this freedom when regular deposits or repayments exist. If a person pays EUR 300 every

month, then $P/Y = 12$ is part of the story. Changing P/Y to 1 would falsely tell the calculator that EUR 300 is paid once per year.

Worked Example 4 - future value using both encodings

EUR 12000 is invested for 4 years at a nominal annual rate of 6%, compounded monthly. Find the future value.

Solution. First let us use Method A. Monthly compounding means 12 periods per year, and because $PMT = 0$ we may let the payment period also be monthly:

$$N = 48, I\% = 6, PV = -12000, PMT = 0, FV = ?, P/Y = C/Y = 12.$$

The solver gives

$$FV = 15245.8699 \dots$$

so the balance is EUR 15245.87.

Now we are going to describe the same investment with Method B. We keep the year as the timeline unit:

$$N = 4, I\% = 6, PV = -12000, PMT = 0, FV = ?, P/Y = 1, C/Y = 12.$$

The calculator returns the same future value. That agreement is a useful check that you understand what N , P/Y and C/Y mean.

Answer: EUR 15245.87

Worked Example 5 - find a present value

A student wants EUR 50000 in 7 years. The bank pays 4.8% nominal interest compounded monthly. How much must be invested now?

Solution. This time the future value is known and the present value is unknown. There are no regular deposits, so $PMT = 0$. Using monthly periods,

$$N = 84, I\% = 4.8, PMT = 0, FV = 50000, P/Y = C/Y = 12.$$

Now we solve for PV . The calculator gives $PV = -35755.1108 \dots$. The minus sign is not a negative amount of money; it tells us that EUR 35755.11 must leave the student's pocket now.

Answer: EUR 35755.11 invested now

Worked Example 6 - solve for the nominal annual rate

EUR 22000 grows to EUR 30000 in 7 years, with interest compounded quarterly. Find the nominal annual interest rate.

Solution. We know both ends of the timeline. Using quarterly periods gives $N = 28$, $P/Y = C/Y = 4$, $PV = -22000$, $FV = 30000$, and $PMT = 0$. Now we are going to ask the solver for $I\%$.

$$I\% = 4.455415 \dots$$

So the nominal annual rate is approximately 4.46%.

IB exam habit. Do not enter 0.0446 in the $I\%$ field. The field already expects a percentage.

Answer: 4.46% nominal p.a., compounded quarterly

Worked Example 7 - solve for time and interpret the decimal

EUR 15000 is invested at 4.2% nominal interest compounded monthly. Find the time required to reach EUR 24000.

Solution. Here we want the number of months, so we use monthly periods:

$$I\% = 4.2, PV = -15000, FV = 24000, PMT = 0, P/Y = C/Y = 12.$$

Solving for N gives $N = 134.5216 \dots$ months. The target is first reached after the next complete

monthly compounding period, so we test month 135. In years, the continuous solution is about 11.21 years.

Answer: about 11.21 years; first complete month 135

3.2 Compounding frequency changes the effective growth

Two banks can quote the same nominal annual rate and still produce different final values if they compound at different frequencies. The more frequently interest is credited, the earlier each small interest payment itself starts earning interest.

Worked Example 8 - annual, half-yearly, quarterly or monthly?

EUR 10000 is invested for 5 years at a nominal annual rate of 5.4%. Compare four compounding frequencies.

Solution. Now we are going to hold the nominal annual rate and time fixed, changing only C/Y . The future values are:

Compounding	Future value (EUR)
annual	13007.78
half-yearly	13052.82
quarterly	13076.00
monthly	13091.71

Monthly compounding gives the largest amount. Notice, however, that the extra gain becomes smaller as the frequency increases. We are not adding huge new interest; we are only changing how quickly already-earned interest begins to compound.

4 Regular deposits: why PMT creates a geometric series

4.1 A savings plan is many small investments layered together

Suppose EUR 500 is deposited at the end of each year. The first deposit has longer to earn interest than the last deposit. At the end of year n , the deposits contribute

$$500R^{n-1} + 500R^{n-2} + \dots + 500R + 500,$$

where $R = 1 + i$ is the growth factor per payment period.

Now we can see the geometric series hiding inside the financial solver. Factor out the deposit amount:

$$500 \left(R^{n-1} + R^{n-2} + \dots + R + 1 \right).$$

The bracket is a finite geometric series.

Future value of an initial amount plus end-of-period deposits

If an amount P is invested now and an additional amount A is deposited at the end of each of n periods, with growth factor $R = 1 + i$, then

$$FV = PR^n + A \frac{R^n - 1}{R - 1}.$$

The TVM solver automates this same geometric-series calculation.

Key insight. This is the connection students often miss. PMT is not mysterious. It represents a geometric series of repeated cash flows, because each payment earns interest for a different number of periods.

Worked Example 9 - initial investment plus annual deposits

An investor places EUR 1000 into an account and then adds EUR 500 at the end of every year for 8 years. The account pays 5% per year, compounded annually. Find the final balance.

Solution. Let us first read the timeline. There is money paid in now and money paid in at the end of each year. Both cash flows leave the investor's pocket, so both are negative in the solver:

$$N = 8, I\% = 5, PV = -1000, PMT = -500, FV = ?, P/Y = C/Y = 1.$$

The solver gives $FV = 6252.0099 \dots$

Now let us connect that answer to geometric series. The initial EUR 1000 grows for 8 years, while the first EUR 500 deposit grows for 7 years and the final deposit grows for 0 years:

$$FV = 1000(1.05)^8 + 500((1.05)^7 + \dots + 1).$$

Both methods are describing the same cash-flow story.

Answer: EUR 6252.01

Worked Example 10 - monthly deposits and monthly compounding

A student invests EUR 2500 now and EUR 180 at the end of every month for 6 years. The nominal annual rate is 4.8%, compounded monthly. Find the final value.

Solution. The repeated payment happens monthly, so the payment period must be monthly. Six years gives $N = 72$. Both the initial EUR 2500 and each EUR 180 deposit leave the student's pocket, so they have the same negative sign:

$$N = 72, I\% = 4.8, PV = -2500, PMT = -180, FV = ?, P/Y = C/Y = 12.$$

Now we evaluate the future value. The solver returns

$$FV = 18317.0878 \dots$$

so the account contains EUR 18317.09.

Answer: EUR 18317.09

Worked Example 11 - annual deposits but monthly compounding

EUR 3000 is invested immediately. A further EUR 700 is added at the end of each year for 10 years. The nominal annual rate is 6%, compounded monthly. Find the final value.

Solution. This is exactly where P/Y and C/Y must be separated. The contribution occurs once per year, so $P/Y = 1$ and $N = 10$. Interest is credited monthly, so $C/Y = 12$.

$$N = 10, I\% = 6, PV = -3000, PMT = -700, FV = ?, P/Y = 1, C/Y = 12.$$

The solver gives EUR 14757.77.

Why is P/Y not 12? Because setting $P/Y = 12$ would tell the calculator that EUR 700 is deposited every month. That would be a completely different savings plan.

Answer: EUR 14757.77

Worked Example 12 - monthly deposits with quarterly compounding

EUR 300 is deposited at the end of each month for 6 years. The nominal annual rate is 6.4%, compounded quarterly. Find the accumulated value.

Solution. The payment story is monthly, so $P/Y = 12$ and $N = 72$. The bank compounds quarterly, so $C/Y = 4$.

$$N = 72, I\% = 6.4, PV = 0, PMT = -300, FV = ?, P/Y = 12, C/Y = 4.$$

The GDC converts the nominal quarterly-compounded rate into the equivalent monthly rate needed for the payment timeline. It returns

$$FV = 26221.1569 \dots$$

so the accumulated value is EUR 26221.16.

Key insight. When $P/Y \neq C/Y$, do not manually divide the nominal annual rate by P/Y . The finance solver handles the conversion between the compounding clock and the payment clock.

Answer: EUR 26221.16

Worked Example 13 - find the monthly deposit needed for a target

A saver already has EUR 12000 and wants EUR 100000 in 10 years. The account pays 4.8% nominal interest compounded monthly. Find the monthly deposit required.

Solution. The existing EUR 12000 and future monthly deposits both leave the saver's pocket, so both are negative. The target is money available later, so FV is positive:

$$N = 120, I\% = 4.8, PV = -12000, PMT = ?, FV = 100000, P/Y = C/Y = 12.$$

Now we solve for PMT . The calculator returns approximately -524.7975 . The negative sign tells us this is a payment made by the saver.

Answer: EUR 524.80 per month

4.2 END versus BEGIN payments

Most IB questions assume end-of-period payments unless stated otherwise. If deposits occur at the beginning of every month, each deposit earns one extra month of interest. Therefore a BEGIN-mode savings plan reaches a higher final value than an otherwise identical END-mode plan.

Trap. Do not switch to BEGIN mode because the phrase "invests now" appears. PV already handles the one-time payment at time 0. BEGIN/END refers specifically to the repeated PMT cash flow.

5 Loans and mortgages

5.1 A loan reverses the cash-flow signs

When you borrow money, the bank gives you money now. From your point of view that is a positive present cash flow, so $PV > 0$. You then repay the bank every month, so $PMT < 0$. If the loan is fully repaid at the end, $FV = 0$.

Now we are going to see why this is still geometric mathematics. The unpaid balance grows by the periodic interest factor, but each repayment then reduces the balance. Repeating that process creates the same type of geometric accumulation that appeared in savings plans, only with opposite cash-flow signs.

Worked Example 14 - monthly mortgage repayment

A borrower takes a EUR 180000 mortgage over 20 years at a nominal annual rate of 4.8%, compounded monthly. Find the monthly repayment.

Solution. There are $20 \times 12 = 240$ monthly repayments, so $N = 240$ and $P/Y = 12$. Compounding is also monthly, so $C/Y = 12$. The borrower receives EUR 180000 now, so $PV = +180000$. The loan is fully cleared at the end, so $FV = 0$.

$$N = 240, I\% = 4.8, PV = 180000, PMT = ?, FV = 0, P/Y = C/Y = 12.$$

Now we solve for the regular cash flow. The calculator gives

$$PMT = -1168.1234 \dots$$

The negative sign is exactly what we expect: every month EUR 1168.12 leaves the borrower's pocket.

Answer: EUR 1168.12 per month

Worked Example 15 - a loan with monthly payments but quarterly compounding

A EUR 120000 loan is repaid monthly over 15 years. The nominal annual rate is 5.6%, compounded quarterly. Find the monthly repayment.

Solution. The repayments are monthly, so $P/Y = 12$ and $N = 180$. The interest is compounded quarterly, so $C/Y = 4$.

$$N = 180, I\% = 5.6, PV = 120000, PMT = ?, FV = 0, P/Y = 12, C/Y = 4.$$

Now the solver converts the quarterly-compounded nominal rate to the equivalent monthly rate for the payment timeline. This is why we must tell it both frequencies. Solving gives a monthly repayment of approximately EUR 985.22.

5.2 Outstanding balance after several years

The outstanding balance is not “original loan minus payments made”. Interest has been accumulating at the same time. There are two clean methods.

Method 1: TVM balance. Keep the original loan, interest rate and repayment, but change N to the number of payments that have occurred and solve for FV .

Method 2: remaining-payments view. Treat the outstanding balance as the present value of all repayments still to come.

Worked Example 16 - outstanding balance after 7 years

For the EUR 180000 mortgage in Worked Example 14, find the outstanding balance immediately after 7 years of payments.

Solution. Seven years means 84 monthly repayments have been made. We keep the original $PV = 180000$, $I\% = 4.8$ and unrounded monthly payment $PMT = -1168.123445\dots$ Now we change only the elapsed number of periods to $N = 84$ and solve for FV .

The calculator gives approximately $FV = -135366.74$. Why negative? At that intermediate time, the balance represents money the borrower still owes the bank; relative to the original positive loan receipt, the remaining obligation is in the opposite direction. The amount still owed is the magnitude: So the amount still owed is **EUR 135366.74**.

Worked Example 17 - total interest on a loan

A EUR 24000 car loan is repaid monthly over 4 years at 6.6% nominal interest compounded monthly.

(a) Find the monthly repayment. (b) Find the total interest paid.

Solution. First we find the repayment:

$$N = 48, I\% = 6.6, PV = 24000, PMT = ?, FV = 0, P/Y = C/Y = 12.$$

This gives $|PMT| = 570.266446\dots$

Now we are going to evaluate the total cash paid over all 48 months. We deliberately use the unrounded payment internally:

$$48(570.266446\dots) = 27372.7894\dots$$

The borrower originally received EUR 24000, so the extra amount is the interest:

$$27372.7894\dots - 24000 = 3372.7894\dots$$

Answer: EUR 570.27 per month; total interest EUR 3372.79

6 Withdrawals and amortization of an investment

When money is invested now, $PV < 0$. If regular withdrawals are received later, $PMT > 0$. Again, the signs are opposite because the cash flows move in opposite directions.

Worked Example 18 - how many full withdrawals are possible?

EUR 30000 is invested at 5.4% nominal interest compounded monthly. EUR 5000 is withdrawn at the end of each year.

(a) Find the number of full EUR 5000 withdrawals that can be made. (b) Find the final partial withdrawal one year later.

Solution. Let us read the two clocks first. The withdrawal happens once each year, so $P/Y = 1$ and the repeated cash flow is $PMT = +5000$: this is money the investor receives. Interest is compounded monthly, so $C/Y = 12$. The original EUR 30000 leaves the investor's pocket at time 0, so $PV = -30000$. The setup is therefore

$$PV = -30000, \quad PMT = 5000, \quad I\% = 5.4, \quad P/Y = 1, \quad C/Y = 12.$$

Now we are going to ask how many complete annual withdrawals the account can support. Using the finance solver, or testing balances year by year, we find that seven full withdrawals are possible. Immediately after the seventh withdrawal, about EUR 2364.37 remains.

That remaining money is not withdrawn immediately. It earns interest for one more year. The effective annual growth factor corresponding to 5.4% nominal interest compounded monthly is

$$\left(1 + \frac{0.054}{12}\right)^{12} = 1.05535675\dots$$

so the final amount available one year later is

$$2364.3658\dots (1.05535675\dots) = 2495.2494\dots$$

Therefore the eighth withdrawal cannot be a full EUR 5000; it is a final partial withdrawal of EUR 2495.25.

IB exam habit. In withdrawal questions, a decimal value of N is not itself the number of full withdrawals. The financial story is discrete. Test the neighbouring whole payment counts and then deal separately with any final partial amount.

Answer: 7 full withdrawals; final partial withdrawal EUR 2495.25

6.1 Why BEGIN and END really matter

A repeated payment made at the beginning of a period has one extra period to earn interest compared with the same payment made at the end. This is not a small calculator setting; it changes the timing of every cash flow.

Worked Example 19 - END payments versus BEGIN payments

A saver deposits EUR 200 every month for 5 years into an account paying 4.8% nominal interest compounded monthly. Compare the final value when deposits are made at the end of each month with the final value when they are made at the beginning.

Solution. In both cases $N = 60$, $I\% = 4.8$, $PV = 0$, $PMT = -200$ and $P/Y = C/Y = 12$. The only change is the timing setting.

With END payments, the final value is the ordinary annuity value. With BEGIN payments, every EUR 200 has one additional month of growth. Therefore the BEGIN value must equal the END value multiplied by one monthly factor, $1 + 0.048/12 = 1.004$.

Now we are going to calculate the two values. The END-mode balance is approximately EUR 13532.04, and the BEGIN-mode balance is approximately

$$13532.04(1.004) \approx \text{EUR } 13586.16.$$

The difference is entirely caused by timing.

Key insight. “Invests EUR 200 at the beginning of every month” changes the *PMT* timing. “Invests EUR 5000 now” is simply a *PV* and does not change the *PMT* timing.

7 Inflation and real value

7.1 Nominal money and purchasing power are different

Suppose an investment grows by 6% per year. If prices also rise by 3% per year, the account balance grows, but each euro buys less. The exact real growth factor is obtained by comparing the two multipliers.

Exact real-value relationship

If the investment grows by factor $1 + i$ per period and prices grow by factor $1 + a$, then

$$\text{Real value after } n \text{ periods} = PV \left(\frac{1 + i}{1 + a} \right)^n.$$

Equivalently,

$$\text{Real value} = \frac{\text{nominal future value}}{(1 + a)^n}.$$

Trap. Simply subtracting the two percentage rates is only an approximation. The exact process is multiplicative, so the correct factor is a ratio.

Worked Example 20 - nominal value versus real value

EUR 40000 is invested for 12 years at 5.5% per year. Inflation averages 2.5% per year.

Solution. First we calculate the nominal amount:

$$40000(1.055)^{12} = 76048.2994 \dots$$

Now we are going to remove the effect of rising prices by dividing by the inflation multiplier:

$$\text{Real value} = \frac{76048.2994 \dots}{(1.025)^{12}} = 56546.1606 \dots$$

The account statement says about EUR 76048, but in today's purchasing power that is only about EUR 56546.

Answer: nominal EUR 76048.30; real EUR 56546.16

Worked Example 21 - a monthly real-value threshold

An investment grows by 0.9% per month while inflation is 0.4% per month. EUR 50000 is invested. Find the first month in which its real value exceeds EUR 60000.

Solution. The exact real monthly factor is

$$R = \frac{1.009}{1.004}.$$

Now we are going to solve the threshold inequality

$$50000R^n > 60000.$$

A logarithm or GDC gives $n > 36.701\dots$, so the first complete month is month 37.

Answer: 37 months

8 Rates that change over time

A standard TVM screen assumes one interest-rate regime. If the rate changes, split the timeline into stages. The final value from stage 1 becomes the starting value for stage 2.

Worked Example 22 - two successive interest regimes

EUR 10000 earns 4% annually for the first 3 years and 5.5% annually for the next 5 years. Find the final value.

Solution. We cannot enter two different interest rates into one ordinary TVM calculation. So we now split the story.

After the first stage,

$$V_3 = 10000(1.04)^3.$$

That amount then becomes the starting balance for the second stage:

$$FV = 10000(1.04)^3(1.055)^5 = 14701.5226\dots$$

Answer: EUR 14701.52

Worked Example 23 - refinancing after several years

A borrower takes a EUR 180000 loan at 5.1% nominal interest compounded monthly over 15 years. After 5 years, the remaining balance is refinanced over the next 10 years at 4.2% nominal interest compounded monthly. Find the original repayment, the balance after 5 years and the new repayment.

Solution. First we are going to solve the *original* loan. The borrower receives EUR 180000 now and makes 180 monthly repayments:

$$N = 180, \quad I\% = 5.1, \quad PV = 180000, \quad FV = 0, \quad P/Y = C/Y = 12.$$

The solver gives $PMT = -1432.822710\dots$, so the original repayment is EUR 1432.82 per month.

Now we move five years along the same timeline. Sixty payments have occurred. We keep the original rate, original PV and the *unrounded* payment, set $N = 60$, and solve for the intermediate future value. The magnitude of the remaining debt is

$$\text{EUR } 134467.92.$$

At this point the first loan story ends. The remaining debt becomes the amount borrowed in a *new* loan, so it is the new positive PV . The new loan lasts 10 years at 4.2%:

$$N = 120, \quad I\% = 4.2, \quad PV = 134467.9238\dots, \quad FV = 0, \quad P/Y = C/Y = 12.$$

Solving gives a new monthly repayment of EUR 1374.24.

Key insight. A refinancing question is two loans joined together. Finish the first stage, carry its remaining balance into the second stage, and never try to put two interest rates into one ordinary TVM screen.

Answer: original EUR 1432.82; balance EUR 134467.92; new payment EUR 1374.24

9 GDC strategy, traps and high-value habits

Key insight: N follows the payment clock. If payments are monthly for 8 years, $N = 96$ even if interest compounds quarterly.

Key insight: C/Y follows the interest clock. Quarterly compounding means $C/Y = 4$ regardless of how often payments are made.

Trap: mixing nominal and periodic rates. The $I\%$ field is normally the nominal annual rate from the question. Do not divide it by 12 before entering it if you are also giving the calculator $C/Y = 12$.

Trap: rounding a payment too early. If a monthly repayment is EUR 1168.123445..., use the unrounded value for later balance and total-interest calculations. Round only the final requested answer.

Trap: forgetting the first special year. If depreciation is 15% in year 1 and 11% thereafter, the value after 10 years contains one factor 0.85 and only nine factors 0.89.

Shortcut: use a ratio for real growth. Instead of finding nominal value and inflation separately, you may use $((1+i)/(1+a))^n$ directly.

Exam habit: write the cash-flow sign sentence. A short phrase such as “ $PV > 0$ because the borrower receives the loan now; $PMT < 0$ because repayments leave the borrower” makes your setup easy to check and easy to award marks for.

10 Past-paper patterns: what IB likes to hide

The IB often places the finance mathematics inside a longer story. The difficult part is usually not pressing the solver button; it is deciding which multiplier applies, how many times it applies, or which quantity is being compared.

The following worked examples are original CF Maths reconstructions inspired by public IB/IB Docs paper structures. Numerical values and wording are changed.

Worked Example 24 - adapted depreciation pattern with a special first year

A car is bought for EUR 42000. It loses 18% of its value during the first year and 9% in each later year.
(a) Find its value after the first year. **(b)** Find its value after 12 complete years. **(c)** Find the least complete year n for which the value is below 12% of the original price.

Solution. Part (a) is one multiplier:

$$42000(0.82) = 34440.$$

Now we are going to count the later years carefully. From the end of year 1 to the end of year 12 there are 11 further depreciation steps, so

$$V_{12} = 34440(0.91)^{11} = 12204.4569 \dots$$

For the threshold, 12% of the original price is EUR 5040. We need

$$34440(0.91)^{n-1} < 5040.$$

Solving gives $n > 21.377 \dots$, so the first complete year is $n = 22$. The exponent is $n - 1$ because the first year has already been handled separately.

Answer: EUR 34440; EUR 12204.46; year 22

Worked Example 25 - depreciation and real value in one question

A car costing EUR 36000 depreciates at 13% per year. Separately, EUR 48000 is invested in an account growing by 1.3% per month while inflation is 0.6% per month.

(a) Find the car's value after 8 years. **(b)** Find the first month in which the real value of the investment exceeds EUR 55000.

Solution. The car keeps 87% of its value each year:

$$36000(0.87)^8 = 11815.6202 \dots$$

For the investment, now we are going to compare the monthly growth and inflation multipliers. The

exact real factor is

$$R = \frac{1.013}{1.006} = 1.00695825 \dots$$

so

$$48000R^n > 55000.$$

This gives $n > 19.632 \dots$, hence the first complete month is 20.

Answer: EUR 11815.62; 20 months

11 Section B synthesis: sustained reasoning

These are deliberately longer. Read them as a story first. In a Section B question, the early parts usually build the model that the final parts need.

Worked Example 26 - large Section B: fixed interest versus compound interest

Two students, Maya and Theo, each have EUR 6000.

Maya uses a savings plan that pays a fixed amount of interest at the end of each month. Each monthly interest payment equals 0.30% of the *initial* EUR 6000. The interest is added to the account but does not itself change future monthly interest payments.

Theo invests EUR 6000 in an account paying 2.9% interest compounded annually.

(a) Find Maya's monthly interest and annual interest.

(b) Write a formula M_n for Maya's balance after n complete years.

(c) Find Maya's balance after 5 years.

(d) Write a formula T_n for Theo's balance after n years and find T_5 .

(e) Determine the smallest complete year for which Theo has more money than Maya.

(f) Explain, in terms of sequence type, why Theo eventually overtakes Maya.

Solution. Start with Maya. The phrase "0.30% of the initial investment" means the same euro amount is paid every month:

$$0.003(6000) = 18 \text{ EUR per month.}$$

Over 12 months this is EUR 216 per year. Therefore Maya's balance increases by the same amount each year, so

$$M_n = 6000 + 216n.$$

Now we evaluate the five-year balance:

$$M_5 = 6000 + 216(5) = 7080.$$

Theo's account is different. His interest is a percentage of the changing balance, so his model is geometric:

$$T_n = 6000(1.029)^n.$$

At five years,

$$T_5 = 6921.9447 \dots$$

so Maya is still ahead.

Now we are going to compare the two models rather than compare one isolated year. Solve or table

$$6000(1.029)^n > 6000 + 216n.$$

At $n = 15$, Theo has about EUR 9212.59 while Maya has EUR 9240. At $n = 16$, Theo has about EUR 9479.76 while Maya has EUR 9456. Therefore Theo first moves ahead after 16 complete years.

Why does this happen? Maya's account is arithmetic: the same EUR 216 is added each year. Theo's account is geometric: the increase itself grows as the balance grows. A geometric growth model with ratio above 1 eventually outgrows a linear arithmetic model.

Answer: monthly EUR 18; annual EUR 216; $M_n = 6000 + 216n$; $T_n = 6000(1.029)^n$; crossover year 16

Worked Example 27 - very large geometric-series and financial-solver question

A student invests EUR 5000 now. At the end of each year she deposits a further EUR 600. The account earns 4.5% effective interest per year.

(a) Explain why the contribution made at the end of year 1 earns interest for one more year than the contribution made at the end of year 2.

(b) Show that after n years the future value can be written

$$F_n = 5000(1.045)^n + 600 \frac{(1.045)^n - 1}{0.045}.$$

(c) Find the balance after 12 years.

(d) Give the complete TVM setup that produces the same answer.

(e) Find the smallest complete year for which the balance exceeds EUR 20000.

(f) The bank changes to a nominal annual rate of 4.5% compounded monthly, while the EUR 600 deposits remain annual. State the correct P/Y and C/Y and explain why they are different.

Solution. The first EUR 600 deposit is made earlier, so by the final date it has more time to grow. This unequal growth time is exactly what creates a geometric series.

After n years, the initial EUR 5000 has grown for all n years:

$$5000(1.045)^n.$$

The deposits contribute

$$600(1.045)^{n-1} + 600(1.045)^{n-2} + \dots + 600.$$

Factor out 600 and apply the finite geometric-series formula:

$$600 \left(1 + (1.045) + \dots + (1.045)^{n-1} \right) = 600 \frac{(1.045)^n - 1}{1.045 - 1}.$$

Therefore

$$F_n = 5000(1.045)^n + 600 \frac{(1.045)^n - 1}{0.045}.$$

Now we are going to evaluate the model at $n = 12$. This gives approximately EUR 17757.83.

The financial solver tells the same story with

$$N = 12, I\% = 4.5, PV = -5000, PMT = -600, FV = ?, P/Y = C/Y = 1.$$

Testing the geometric model or using a GDC gives $F_{13} < 20000$ and $F_{14} = 20618.99 \dots > 20000$, so the first complete year is $n = 14$.

Finally, if compounding becomes monthly while deposits remain annual, the payment clock is still annual, so $P/Y = 1$. The interest clock is monthly, so $C/Y = 12$. They are different because they describe different events.

Answer: $F_{12} \approx \text{EUR } 17757.83$; first exceeds EUR 20000 in year 14; new frequencies $P/Y = 1$, $C/Y = 12$

Worked Example 28 - past-paper-style Section B: a growing prize, an investment and an unknown rate

Leila and Omar each receive a large sum of money in different ways.

Leila is offered two ways to receive a prize over 30 months. Under Plan A she receives EUR 3200 at the beginning of each month. Under Plan B she receives EUR 1400 in the first month and each later monthly payment is 5% greater than the previous payment.

Omar invests EUR 90000 in an account paying 4.2% nominal annual interest compounded monthly.

(a) Find the total amount Leila receives under each plan after 30 payments.

(b) Write a formula for the value of Omar's investment after m months.

(c) Suppose Leila chooses Plan B and keeps each payment as cash. Find the combined value of Leila's received payments and Omar's investment at the end of month 8.

(d) Find the first complete month in which their combined value is at least EUR 175000.

(e) At the end of the 30 months, Leila invests EUR 35000 for 5 years at a nominal annual rate of $r\%$, compounded quarterly. The investment grows to EUR 46000. Find r .

Solution. Part (a) deliberately contains two different sequence stories. Plan A is constant:

$$30(3200) = 96000.$$

Plan B is geometric. The first payment is 1400, the common ratio is 1.05 and there are 30 payments:

$$S_{30} = 1400 \frac{1.05^{30} - 1}{1.05 - 1} = 93014.3865 \dots$$

So Plan A pays more in total over these 30 months, even though Plan B is growing.

For Omar, the monthly periodic factor is $1 + 0.042/12 = 1.0035$. Therefore after m months,

$$O_m = 90000(1.0035)^m.$$

Now we are going to combine two quantities at the same date. By the end of month 8, Leila has received the first eight Plan B payments:

$$L_8 = 1400 \frac{1.05^8 - 1}{0.05} = 13368.7524 \dots$$

Omar's investment is

$$O_8 = 90000(1.0035)^8 = 92551.0870 \dots$$

The combined value is therefore EUR 105919.84.

For part (d), we should not round either model. We solve or table

$$1400 \frac{1.05^m - 1}{0.05} + 90000(1.0035)^m \geq 175000.$$

At month 26 the total is about EUR 170117.39; at month 27 it is about EUR 175440.28. Hence the first complete month is 27.

Finally, the quarterly investment has $5 \times 4 = 20$ compounding periods:

$$46000 = 35000 \left(1 + \frac{r/100}{4}\right)^{20}.$$

Now we solve for the nominal annual percentage rate:

$$r = 5.50338 \dots \%$$

Key insight. This is a typical sustained-response structure: geometric-series sum, compound growth, combining values at one date, a discrete threshold, then solving backwards for a rate. Do not try to solve the whole page in one calculator screen.

Answer: Plan A EUR 96000; Plan B EUR 93014.39; month 8 total EUR 105919.84; threshold month 27; $r \approx 5.50\%$

12 Final strategy sheet

A reliable decision order

- 1. Tell the story.** What happens at time 0? What happens repeatedly? What is wanted at the end?
- 2. Identify the clocks.** What is the payment frequency? What is the compounding frequency?
- 3. Set the signs.** Money entering your pocket and money leaving your pocket must have opposite signs.

4. **Set N .** N counts payment periods, not automatically compounding periods.
5. **Only now use the formula or GDC.**
6. **Interpret the output.** A negative PMT is a payment made; a decimal N may need a whole-period interpretation; an intermediate loan FV may be negative because it is still owed.
7. **Keep full precision until the final answer.**

Quick sign setups

Savings: $PV < 0$, deposits $PMT < 0$, target $FV > 0$.

Loan: $PV > 0$, repayments $PMT < 0$, fully repaid $FV = 0$.

Investment with withdrawals: $PV < 0$, withdrawals $PMT > 0$.

Quick frequency setups

Annual payment, monthly compounding: $P/Y = 1$, $C/Y = 12$.

Monthly payment, quarterly compounding: $P/Y = 12$, $C/Y = 4$.

No PMT : either use $P/Y = C/Y = k$ with $N = kt$, or keep $P/Y = 1$, $C/Y = k$, $N = t$.

Source and scope note. This chapter follows IB Mathematics: Analysis and Approaches SL 1.4, where financial applications of geometric sequences and series include compound interest, annual depreciation, real value under inflation and technology including built-in financial packages. Past-paper structures were reviewed against the public IB/IB Docs archive, including the 2021 AA specimen finance question, the May 2023 AA SL finance question, the November 2023 AA SL extended-response finance-and-geometric-series question, the May 2024 AA HL depreciation pattern, and the May 2025 AA HL finance/real-value pattern. Questions and worked examples here are original or materially reconstructed rather than reproductions of official wording.