

CF MATHS

IB Mathematics AA

Financial Applications - Detailed Worked Solutions

Words before buttons. Timeline before arithmetic. Full precision before rounding.

These solutions are deliberately written as teaching notes rather than as short mark schemes. For each financial problem we explain what is happening to the money, why the signs have the direction they do, which clock determines N and P/Y , which clock determines C/Y , and only then carry out the calculation.

Section A - Growth, depreciation and lump sums

Solution 1

Each year the account grows by 3.8%, so the multiplier is

$$1 + \frac{3.8}{100} = 1.038.$$

That is the common ratio. Now we are going to apply the multiplier nine times:

$$12500(1.038)^9 = 17485.8277 \dots$$

The model is geometric because the same *multiplier* is repeated. The euro increase is not constant; it becomes larger as the balance becomes larger.

Answer: common ratio 1.038; balance EUR 17485.83.

Solution 2

A 12% loss means the machine keeps 88% of its value each year, so the multiplier is 0.88.

For six years,

$$V_6 = 68000(0.88)^6 = 31579.4779 \dots,$$

so the value is EUR 31579.48.

For the threshold, we solve

$$68000(0.88)^n < 30000.$$

A logarithm or a GDC gives $n > 6.401 \dots$. The question asks for a complete year, so now we interpret the decimal: after 6 years the value is still above EUR 30000, and after 7 it is below. Therefore the first complete year is 7.

Answer: EUR 31579.48; year 7.

Solution 3

We know the final value and want the original value. Depreciating by 6% means multiplying by 0.94 each year. If the original price is P ,

$$P(0.94)^5 = 28600.$$

Now solve backwards:

$$P = \frac{28600}{0.94^5} = 38969.6733 \dots$$

Answer: original price EUR 38969.67.

Solution 4

The unknown is the nominal annual rate. Because compounding is monthly and there are no repeated deposits, one clean setup is

$$N = 72, \quad PV = -18000, \quad PMT = 0, \quad FV = 25000, \quad P/Y = C/Y = 12.$$

Why opposite signs? The EUR 18000 leaves the investor now, whereas EUR 25000 is money received later. Now solve for $I\%$. The calculator gives

$$I\% = 5.4875769 \dots$$

Answer: 5.49% nominal per year, compounded monthly.

Solution 5

Quarterly compounding means the periodic rate is $4.6\%/4 = 1.15\%$ per quarter. We solve

$$24000(1.0115)^N > 40000.$$

The boundary is

$$N = 44.6745 \dots \text{ quarters.}$$

The balance only updates at complete compounding periods, so the first valid quarter is 45. This corresponds to $45/4 = 11.25$ years.

Answer: 45 complete quarters, or 11.25 years.

Solution 6

For annual compounding,

$$FV = 10000(1.052)^8 = 15001.1971 \dots$$

For monthly compounding, the monthly rate is $0.052/12$ and there are 96 months:

$$FV = 10000 \left(1 + \frac{0.052}{12} \right)^{96} = 15145.2410 \dots$$

The extra amount is

$$15145.2410 - 15001.1971 = 144.0438 \dots$$

It is positive because interest is credited earlier under monthly compounding, so some of that interest begins earning further interest sooner.

Answer: annual EUR 15001.20; monthly EUR 15145.24; extra EUR 144.04.

Solution 7

The first year is special. The vehicle keeps 80%:

$$45000(0.80) = 36000.$$

After that it keeps 92.5% per year. From the end of year 1 to the end of year 10 there are nine later depreciation steps, so

$$V_{10} = 36000(0.925)^9 = 17847.5290 \dots$$

For 30% of the original price, the target is EUR 13500. For $n \geq 1$,

$$V_n = 36000(0.925)^{n-1}.$$

Solve $36000(0.925)^{n-1} < 13500$. The boundary is $n > 13.5809 \dots$, so the first complete year is 14.

Trap. Do not use ten factors of 0.925 in part (b). One year has already been handled by the 20% first-year depreciation.

Answer: EUR 36000; EUR 17847.53; year 14.

Section B - TVM and regular savings

Solution 8

There are three cash-flow directions to interpret. The EUR 9000 initial investment leaves the student's pocket now, so $PV = -9000$. Each EUR 250 contribution also leaves the student's pocket, so $PMT = -250$. The final balance is money the student can receive later, so $FV > 0$.

Monthly payments for 5 years give $N = 60$ and $P/Y = 12$. Interest is compounded monthly, so $C/Y = 12$:

$$N = 60, I\% = 4.2, PV = -9000, PMT = -250, FV = ?, P/Y = C/Y = 12.$$

Now we solve for the future value. The calculator gives

$$FV = 27758.0196 \dots$$

Answer: EUR 27758.02.

Solution 9

Because $PMT = 0$, we can describe the same lump-sum investment using two different payment clocks.

Monthly-period encoding:

$$N = 36, I\% = 6, PV = -20000, PMT = 0, FV = ?, P/Y = 12, C/Y = 12.$$

Annual-period encoding:

$$N = 3, I\% = 6, PV = -20000, PMT = 0, FV = ?, P/Y = 1, C/Y = 12.$$

Both describe 3 years of monthly compounding and both give

$$FV = 23933.6105 \dots$$

If EUR 400 were deposited every month, however, $P/Y = 12$ would be part of the actual cash-flow story. Setting $P/Y = 1$ would incorrectly tell the calculator that the EUR 400 payment happens once per year.

Answer: both setups give EUR 23933.61.

Solution 10

The EUR 4000 invested now grows for all nine years:

$$4000(1.051)^9.$$

The first EUR 750 end-of-year deposit grows for 8 years, the next for 7, and the last for 0 years. Therefore

$$FV = 4000(1.051)^9 + 750((1.051)^8 + (1.051)^7 + \dots + 1).$$

Now use the finite geometric-series sum:

$$FV = 4000(1.051)^9 + 750 \frac{1.051^9 - 1}{0.051} = 14562.7644 \dots$$

Answer: EUR 14562.76.

Solution 11

The payments and compounding are both monthly. Seven years gives $N = 84$:

$$N = 84, I\% = 5.4, PV = 0, PMT = -250, FV = ?, P/Y = C/Y = 12.$$

The negative payment means EUR 250 leaves the saver every month. Solving gives

$$FV = 25451.4439 \dots$$

Answer: EUR 25451.44.

Solution 12

The repeated contribution happens annually, so the payment clock is annual: $N = 12$ and $P/Y = 1$. Interest is compounded monthly, so $C/Y = 12$.

$$N = 12, I\% = 6, PV = -5000, PMT = -800, FV = ?, P/Y = 1, C/Y = 12.$$

Now the finance solver converts the nominal monthly-compounded rate into the equivalent annual rate for the annual payment timeline. It gives

$$FV = 23882.6528 \dots$$

Setting $P/Y = 12$ would mean EUR 800 is paid every month, which is not the problem given.

Answer: $N = 12, P/Y = 1, C/Y = 12$; EUR 23882.65.

Solution 13

Monthly deposits for 8 years give $N = 96$ and $P/Y = 12$. Quarterly compounding gives $C/Y = 4$:

$$N = 96, I\% = 6.2, PV = 0, PMT = -300, FV = ?, P/Y = 12, C/Y = 4.$$

We now let the financial package reconcile the two clocks. The accumulated value is

$$FV = 37113.5494 \dots$$

Answer: EUR 37113.55.

Solution 14

The saver already has money in the account and will add more money every month. From the saver's point of view both are outflows, so

$$PV = -10000, \quad PMT < 0,$$

while the target is money available later, so $FV = 80000 > 0$.

Nine years of monthly payments gives

$$N = 108, I\% = 4.8, PV = -10000, PMT = ?, FV = 80000, P/Y = C/Y = 12.$$

Solving gives $PMT = -479.4724 \dots$. The negative sign is direction; the amount deposited is EUR 479.47.

Answer: EUR 479.47 per month.

Solution 15

For END payments,

$$N = 60, I\% = 4.5, PV = 0, PMT = -220, FV = ?, P/Y = C/Y = 12,$$

and the calculator gives EUR 14772.02.

For BEGIN payments all fields are the same except the timing setting. Each payment earns one additional month of interest, giving EUR 14827.42.

The second value is larger because every deposit is made one month earlier. The difference is not caused by a different interest rate; it is caused entirely by the timing of the repeated cash flows.

Answer: END EUR 14772.02; BEGIN EUR 14827.42.

Section C - Loans, mortgages and withdrawals

Solution 16

A loan reverses the usual savings signs. The borrower receives EUR 250000 now, so $PV = +250000$. The monthly repayments leave the borrower, so the calculator should return a negative PMT . A fully repaid loan has $FV = 0$.

$$N = 25(12) = 300, \quad I\% = 4.5, \quad PV = 250000, \quad FV = 0, \quad P/Y = C/Y = 12.$$

Solving gives

$$PMT = -1389.5812 \dots$$

The repayment amount is therefore EUR 1389.58 per month.

Answer: EUR 1389.58 per month.

Solution 17

Payments occur monthly for 12 years, so $N = 144$ and $P/Y = 12$. Interest compounds quarterly, so $C/Y = 4$:

$$N = 144, \quad I\% = 5.4, \quad PV = 135000, \quad PMT = ?, \quad FV = 0, \quad P/Y = 12, \quad C/Y = 4.$$

The solver gives $PMT = -1274.2099 \dots$, so the repayment is EUR 1274.21.

Why not divide 5.4% by 12? Because the quoted nominal rate is compounded quarterly, not monthly. The finance solver uses $C/Y = 4$ to interpret that nominal rate and then converts it to the equivalent monthly payment-period rate.

Answer: $N = 144, P/Y = 12, C/Y = 4$; EUR 1274.21 per month.

Solution 18

We return to the mortgage in Question 16 and keep the unrounded payment 1389.581194...

Seven years means 84 payments have been made. One method is to keep the original PV , rate and payment, change N to 84, and solve for the intermediate FV . The magnitude of the remaining debt is

$$205460.6886 \dots$$

So the outstanding balance is EUR 205460.69.

For total interest, there are 300 payments. Use the unrounded payment:

$$300(1389.581194 \dots) = 416874.3585 \dots$$

Subtract the EUR 250000 originally borrowed:

$$416874.3585 \dots - 250000 = 166874.3585 \dots$$

Answer: balance EUR 205460.69; total interest EUR 166874.36.

Solution 19

The loan setup is

$$N = 60, \quad I\% = 7.2, \quad PV = 32000, \quad PMT = ?, \quad FV = 0, \quad P/Y = C/Y = 12.$$

The solver gives $PMT = -636.662234 \dots$, hence a repayment of EUR 636.66.

Now we are going to evaluate the total of all 60 payments using the unrounded value:

$$60(636.662234 \dots) = 38199.7341 \dots$$

Therefore the total interest is

$$38199.7341 \dots - 32000 = 6199.7341 \dots$$

Answer: EUR 636.66 per month; total repaid EUR 38199.73; interest EUR 6199.73.

Solution 20

The EUR 45000 is invested now, so $PV = -45000$. Each EUR 6000 withdrawal is money received by the investor, so $PMT = +6000$. The payment clock is annual, $P/Y = 1$, while the compounding clock is monthly, $C/Y = 12$.

The effective yearly factor produced by 5.2% nominal monthly compounding is

$$R = \left(1 + \frac{0.052}{12}\right)^{12} = 1.05325741 \dots$$

Testing complete annual withdrawals, nine full EUR 6000 withdrawals are possible. Immediately after the ninth withdrawal, EUR 4729.64 remains. Let that balance grow for one more year:

$$4729.6440 \dots R = 4981.5326 \dots$$

That is the final partial withdrawal.

Answer: $PV < 0$, $PMT > 0$; 9 full withdrawals; final partial withdrawal EUR 4981.53.

Solution 21

First solve the original 20-year mortgage:

$$N = 240, I\% = 5.0, PV = 200000, PMT = ?, FV = 0, P/Y = C/Y = 12.$$

This gives

$$PMT = -1319.911478 \dots,$$

so the original repayment is EUR 1319.91.

After 6 years, 72 payments have been made. Keeping the original rate and unrounded payment, the remaining debt is

$$159242.1424 \dots$$

This amount now becomes the positive PV of the new 14-year loan:

$$N = 168, I\% = 4.1, PV = 159242.1424 \dots, PMT = ?, FV = 0, P/Y = C/Y = 12.$$

The new repayment is EUR 1247.37.

Now compare total payments. Original plan:

$$240(1319.911478 \dots) = 316778.75 \text{ EUR approximately.}$$

Refinancing plan, ignoring fees:

$$72(1319.911478 \dots) + 168(1247.366508 \dots) = 304591.20 \text{ EUR approximately.}$$

The refinancing plan is about EUR 12187.56 cheaper in total under the assumptions of the question.

Answer: old payment EUR 1319.91; balance EUR 159242.14; new payment EUR 1247.37; saving about EUR 12187.56.

Section D - Inflation, real value and changing rates

Solution 22

The nominal value ignores inflation:

$$60000(1.061)^{15} = 145841.7977 \dots$$

Now we are going to convert this future money into today's purchasing power. Prices have multiplied by $(1.024)^{15}$, so

$$\text{real value} = \frac{145841.7977 \dots}{1.024^{15}} = 102183.9436 \dots$$

The exact real annual multiplier is

$$\frac{1.061}{1.024} = 1.0361328125,$$

so the exact real growth rate is 3.61328125% per year.

Subtracting $6.1 - 2.4 = 3.7\%$ treats two multiplicative processes as if they were additive. It is a useful approximation when rates are small, but it is not exact.

Answer: nominal EUR 145841.80; real EUR 102183.94; exact real rate 3.613% p.a.

Solution 23

Investment money grows by factor 1.01 per month while prices grow by factor 1.0045. The exact real monthly multiplier is therefore

$$R = \frac{1.01}{1.0045} = 1.0054753609 \dots$$

We solve

$$50000R^n > 65000.$$

The boundary is $n > 48.0483 \dots$. At month 48 the real value is about EUR 64982.85, still below the target; at month 49 it is about EUR 65338.66. Therefore month 49 is the first complete month.

Answer: real multiplier 1.00547536...; 49 months.

Solution 24

The rate changes, so the timeline must be split into two stages. For the first 4 years,

$$V_4 = 18000(1.036)^4.$$

That value becomes the starting balance for the next 7 years:

$$FV = 18000(1.036)^4(1.052)^7 = 29567.9834 \dots$$

An ordinary TVM solver screen contains only one interest-rate field, so it cannot model both rate regimes simultaneously. We finish stage 1 first and carry its ending value into stage 2.

Answer: EUR 29567.98.

Solution 25

Again the timeline has two stages. The first stage is 3 years at 4.8% nominal compounded monthly:

$$V_3 = 25000 \left(1 + \frac{0.048}{12} \right)^{36}.$$

Now we use that amount as the starting value for 4 further years at 3.5% nominal compounded quarterly:

$$FV = V_3 \left(1 + \frac{0.035}{4} \right)^{16} = 33181.0736 \dots$$

Answer: EUR 33181.07.

Solution 26

The account begins with an investment and then receives monthly deposits, so from the saver's perspective

$$PV = -8000, \quad PMT = -220, \quad FV = 40000.$$

Nine years of monthly payments gives $N = 108$, and both payment and compounding clocks are monthly:

$$N = 108, \quad I\% = ?, \quad PV = -8000, \quad PMT = -220, \quad FV = 40000, \quad P/Y = C/Y = 12.$$

Now we solve for the nominal annual rate. The calculator returns

$$I\% = 3.998779 \dots$$

Answer: approximately 4.00% nominal per year, compounded monthly.

Section E - Very difficult geometric-series financial synthesis

Solution 27

The EUR 600 deposits are made at different times. The first deposit is in the account for many more years than the last. When every contribution is transported to the same final date, the interest factors are successive powers of 1.045, which is exactly a geometric pattern.

The initial EUR 5000 grows for all n years:

$$5000(1.045)^n.$$

The end-of-year deposits contribute

$$600(1.045)^{n-1} + 600(1.045)^{n-2} + \dots + 600.$$

Now factor out 600 and apply the finite geometric-series formula:

$$600 \left(1 + 1.045 + \dots + 1.045^{n-1} \right) = 600 \frac{1.045^n - 1}{1.045 - 1}.$$

Hence

$$F_n = 5000(1.045)^n + 600 \frac{1.045^n - 1}{0.045}.$$

At $n = 12$,

$$F_{12} = 17757.8263 \dots,$$

so the balance is EUR 17757.83.

The equivalent TVM setup is

$$N = 12, I\% = 4.5, PV = -5000, PMT = -600, FV = ?, P/Y = C/Y = 1.$$

For the threshold, $F_{13} = 19156.93 \dots < 20000$, while

$$F_{14} = 20618.99 \dots > 20000.$$

Therefore the first complete year is 14.

If compounding changes to monthly but deposits remain annual, then the payment clock is still $P/Y = 1$ and the interest clock becomes $C/Y = 12$.

Key insight. The geometric-series formula and the TVM solver are not competing methods. They are two descriptions of the same repeated-payment accumulation.

Answer: $F_{12} = \text{EUR } 17757.83$; threshold year 14; new $P/Y = 1, C/Y = 12$.

Section F - Five long past-paper-pattern questions

Solution 28

Quarterly compounding at a nominal annual rate of 5.8% means a quarterly rate of

$$\frac{0.058}{4} = 0.0145.$$

One year contains four of these quarterly multipliers, so the year-to-year common ratio is

$$r = (1.0145)^4 = 1.0592737387 \dots \approx 1.059.$$

Thus the year-end balances are $P, Pr, Pr^2, \dots$, a geometric sequence.

To double, we need

$$Pr^n > 2P \iff r^n > 2.$$

The continuous boundary is

$$n > \frac{\ln 2}{\ln r} = 12.0372 \dots$$

Therefore the first complete year is 13.

If $P = 12500$, the balance at the end of year 13 is

$$12500r^{13} = 26425.1027 \dots$$

Using 1.058 as the annual common ratio would be wrong because 5.8% is a *nominal* annual rate compounded quarterly. The correct annual effective multiplier must apply the quarterly factor four times.

Answer: $r = 1.059$ to 4 s.f.; first doubles in year 13; balance EUR 26425.10.

Solution 29

The model $C(t) = 46000(0.92)^t$ tells us immediately that the car keeps 92% per year, so the depreciation rate is 8%.

After 6 years,

$$C(6) = 46000(0.92)^6 = 27892.3301 \dots$$

Alex's investment after 6 years is

$$18000(1.034)^6 = 21998.6352 \dots$$

Therefore the extra amount required is

$$27892.3301 \dots - 21998.6352 \dots = 5893.6949 \dots$$

For the crossover, compare the two models:

$$18000(1.034)^n > 46000(0.92)^n.$$

The boundary is $n > 8.0320 \dots$. At year 8 Alex has about EUR 23519.97 while the car is worth about EUR 23608.07, so he is still short. At year 9 his investment is about EUR 24319.65 while the car is worth about EUR 21719.42. Hence year 9 is the first complete year.

Answer: 8% depreciation; car EUR 27892.33; investment EUR 21998.64; extra EUR 5893.69; crossover year 9.

Solution 30

The first-year loss is 16%, so the car keeps 84%:

$$V_1 = 48000(0.84) = 40320.$$

There are then ten later 10% depreciation steps from the end of year 1 to the end of year 11:

$$V_{11} = 40320(0.90)^{10} = 14058.7147 \dots$$

For any $n \geq 1$,

$$V_n = 40320(0.90)^{n-1}.$$

The 15% threshold is

$$0.15(48000) = 7200.$$

We solve

$$40320(0.90)^{n-1} < 7200.$$

The boundary is $n > 17.3512 \dots$. To check the integer transition, $V_{17} \approx 7471.38$ is still above the threshold and $V_{18} \approx 6724.24$ is below. Therefore year 18 is first.

Why $n - 1$? The factor 40320 already represents the end of the special first year. Only the remaining $n - 1$ years use the 10% depreciation rule.

Answer: EUR 40320; EUR 14058.71; $V_n = 40320(0.9)^{n-1}$; year 18.

Solution 31

The equipment keeps 86% of its value each year:

$$42000(0.86)^9 = 10807.7515 \dots$$

For the investment, do not subtract rates first. Investment purchasing power is the ratio of the investment multiplier to the price multiplier:

$$R = \frac{1.014}{1.0065} = 1.0074515648 \dots$$

So the exact real monthly growth rate is about 0.745156%.

Now solve

$$55000R^n > 65000.$$

The boundary is $n > 22.5021 \dots$. At month 22 the real value is about EUR 64758.17; at month 23 it is about EUR 65240.72. Therefore the first complete month is 23.

The subtraction method gives $1.4 - 0.65 = 0.75\%$. This is close but not exact because the true model divides multipliers rather than subtracting percentages. Using 1.0075^n gives a boundary of about 22.3573 months, which still rounds to the same first complete month, 23, in this particular case. A different threshold could make the integer answer differ.

Answer: equipment EUR 10807.75; exact real multiplier 1.00745156...; threshold month 23; subtraction approximation also gives month 23 here.

Solution 32

Plan A is constant, so 30 payments give

$$30(3200) = 96000.$$

Plan B is a geometric sequence with first term 1400, common ratio 1.05 and 30 terms:

$$S_{30} = 1400 \frac{1.05^{30} - 1}{0.05} = 93014.3865 \dots$$

Omar's monthly investment factor is

$$1 + \frac{0.042}{12} = 1.0035,$$

so after m months

$$O_m = 90000(1.0035)^m.$$

By the end of month 8, Leila has received eight Plan B payments:

$$L_8 = 1400 \frac{1.05^8 - 1}{0.05} = 13368.7524 \dots$$

Omar's investment is

$$O_8 = 90000(1.0035)^8 = 92551.0870 \dots$$

The combined value is therefore

$$105919.8395 \dots,$$

or EUR 105919.84.

For the threshold, now we combine the two models at the same month:

$$1400 \frac{1.05^m - 1}{0.05} + 90000(1.0035)^m \geq 175000.$$

A GDC table or numerical solve gives about EUR 170117.39 at month 26 and EUR 175440.28 at month 27. Hence the first complete month is 27.

For the final rate problem, 5 years of quarterly compounding gives 20 quarters:

$$46000 = 35000 \left(1 + \frac{r/100}{4} \right)^{20}.$$

Solving,

$$r = 5.503382 \dots$$

Key insight. Notice the sequence of decisions in this long question. First identify a geometric sum, then a compound-growth formula, then add values measured at the same date, then interpret a whole-month threshold, and finally solve a new investment backwards for a rate.

Answer: Plan A EUR 96000; Plan B EUR 93014.39; $O_m = 90000(1.0035)^m$; month 8 total EUR 105919.84; threshold month 27; $r = 5.50\%$.

Past-paper source note. These final solutions correspond to materially reconstructed question structures reviewed from the public IB/IB Docs archive. They are written as original CF Maths teaching material rather than reproductions of official questions or markschemes.