

CF MATHS

IB Mathematics AA

Financial Applications Worksheet

Compound interest • TVM • annuities • loans • inflation • geometric series

How to use this worksheet

Questions run continuously from 1 to 32. Start by translating every story into a timeline and, where a TVM solver is used, write the values of N , $I\%$, PV , PMT , FV , P/Y and C/Y before calculating. The final section contains five long, materially reconstructed IB past-paper structures. They are not reproductions of official wording.

Expected standard: medium through very difficult. Full working and interpretation are expected. Keep unrounded calculator values for intermediate steps.

Section A - Growth, depreciation and lump sums

1. EUR 12500 is invested at 3.8% effective interest per year. (a) State the common ratio of the sequence of year-end balances. [1] (b) Find the balance after 9 years. [2] (c) Explain why the balances form a geometric rather than arithmetic sequence. [1]
2. A machine is purchased for EUR 68000 and depreciates by 12% each year. (a) Find its value after 6 years. [2] (b) Find the first complete year in which its value is below EUR 30000. [3]
3. A boat is sold for EUR 28600 after depreciating by 6% per year for 5 years. Find its original price. [4]
4. **GDC** EUR 18000 grows to EUR 25000 in 6 years in an account compounded monthly. Find the nominal annual interest rate. [4]
5. **GDC** EUR 24000 is invested at 4.6% nominal annual interest compounded quarterly. Find the first complete quarter in which the balance exceeds EUR 40000, and give the corresponding time in years. [5]
6. EUR 10000 is invested for 8 years at a nominal annual rate of 5.2%. (a) Find the final value if interest is compounded annually. [2] (b) Find the final value if interest is compounded monthly. [2] (c) Find the extra amount obtained from monthly compounding and explain why it is positive. [2]
7. A vehicle costs EUR 45000. It loses 20% of its value during the first year and 7.5% in each later year. (a) Find its value after the first year. [1] (b) Find its value after 10 complete years. [3] (c) Find the first complete year in which its value is below 30% of the original price. [4]

Section B - TVM and regular savings

8. **GDC** A student invests EUR 9000 now and EUR 250 at the end of every month for 5 years. The nominal annual rate is 4.2%, compounded monthly. (a) Explain the signs of PV , PMT and FV . [3] (b) Write the complete TVM setup. [3] (c) Find the final value. [2]
9. **GDC** EUR 20000 is invested for 3 years at 6% nominal annual interest compounded monthly, with no regular deposits. (a) Give a TVM setup using monthly payment periods. [2] (b) Give a different valid TVM setup using annual payment periods. [2] (c) Verify that both produce the same final value. [2] (d) Explain why this freedom would not be valid if EUR 400 were deposited every month. [2]
10. An investor places EUR 4000 into an account now and EUR 750 at the end of each year for 9 years. The account earns 5.1% effective interest per year. (a) Write the final balance as the sum of the growth of the initial EUR 4000 and a finite geometric series. [4] (b) Find the final balance. [2]
11. **GDC** EUR 250 is deposited at the end of every month for 7 years at 5.4% nominal interest compounded monthly. Find the accumulated value. [4]
12. **GDC** EUR 5000 is invested now and EUR 800 is added at the end of every year for 12 years. Interest is 6% nominal compounded monthly. (a) State N , P/Y and C/Y . [3] (b) Find the final value. [3] (c) Explain why setting $P/Y = 12$ would represent a different financial story. [2]
13. **GDC** EUR 300 is deposited at the end of every month for 8 years. The nominal annual rate is 6.2%, compounded quarterly. (a) State N , P/Y and C/Y . [3] (b) Find the final value. [3]
14. **GDC** A saver already has EUR 10000 and wants EUR 80000 in 9 years. The account pays 4.8% nominal interest compounded monthly. Find the required end-of-month deposit. [5]
15. **GDC** EUR 220 is deposited every month for 5 years at 4.5% nominal interest compounded monthly. (a) Find the final value for end-of-month deposits. [3] (b) Find the final value for beginning-of-month deposits. [3] (c) Explain why the second answer is larger. [2]

Section C - Loans, mortgages and withdrawals

16. **GDC** A EUR 250000 mortgage is repaid monthly over 25 years at 4.5% nominal annual interest compounded monthly. Find the monthly repayment and explain the sign of the calculator output. [5]
17. **GDC** A EUR 135000 loan is repaid monthly over 12 years at 5.4% nominal interest compounded quarterly. (a) State N , P/Y and C/Y . [3] (b) Find the monthly repayment. [3] (c) Explain why dividing 5.4% by 12 manually is not the correct setup. [2]
18. For the mortgage in Question 16: (a) find the outstanding balance immediately after 7 years of payments; [4] (b) find the total interest paid if all scheduled payments are made. [3]
19. **GDC** A EUR 32000 car loan is repaid monthly over 5 years at 7.2% nominal interest compounded monthly. (a) Find the monthly repayment. [3] (b) Find the total amount repaid. [2] (c) Hence find the total interest. [2]
20. **GDC** EUR 45000 is invested at 5.2% nominal interest compounded monthly. EUR 6000 is withdrawn at the end of each year. (a) State the signs of PV and PMT . [2] (b) Find the number of full EUR 6000 withdrawals possible. [4] (c) Find the final partial withdrawal one year after the last full withdrawal. [4]
21. **GDC** A EUR 200000 mortgage is arranged over 20 years at 5.0% nominal interest compounded monthly. (a) Find the original monthly repayment. [3] (b) Find the balance after 6 years. [4] (c) The balance is refinanced over the remaining 14 years at 4.1% nominal interest compounded monthly. Find the new repayment. [3] (d) Ignoring fees, compare the total amount paid under the original plan with the refinancing plan. [4]

Section D - Inflation, real value and changing rates

22. EUR 60000 is invested for 15 years at 6.1% effective annual interest while inflation averages 2.4% per year. (a) Find the nominal future value. [2] (b) Find the real value in today's euros. [3] (c) Find the exact effective annual real growth rate and explain why $6.1 - 2.4 = 3.7\%$ is not exact. [3]
23. An investment of EUR 50000 grows by 1.0% per month while inflation is 0.45% per month. Find the first complete month in which the real value exceeds EUR 65000. Show the real monthly multiplier used. [5]
24. EUR 18000 earns 3.6% effective annual interest for 4 years and then 5.2% effective annual interest for the next 7 years. Find the final value and explain why one ordinary TVM calculation cannot use both rates simultaneously. [4]
25. EUR 25000 is invested for 3 years at 4.8% nominal interest compounded monthly, then for 4 further years at 3.5% nominal interest compounded quarterly. Find the final value. [5]
26. **GDC** An account starts with EUR 8000 and receives EUR 220 at the end of every month for 9 years. The final value is EUR 40000. Interest is compounded monthly. Find the nominal annual interest rate. [6]

Section E - Very difficult geometric-series financial synthesis

27. A student invests EUR 5000 now and deposits EUR 600 at the end of each year. The account earns 4.5% effective annual interest. (a) Explain why the yearly deposits form a geometric series when viewed at the final date. [2] (b) Show that the balance after n years is

$$F_n = 5000(1.045)^n + 600 \frac{(1.045)^n - 1}{0.045}.$$

[5] (c) Find F_{12} . [2] (d) Give the complete TVM setup that gives the same result. [3] (e) Find the first complete year for which the balance exceeds EUR 20000. [3] (f) The bank changes to 4.5% nominal interest compounded monthly while deposits remain annual. State the new P/Y and C/Y , explaining why they differ. [3]

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Section F - Five long past-paper-pattern questions

These five questions are deliberately long. Their structures were reconstructed from IB/IB Docs papers; names, values, wording and extensions are changed substantially.

28. PAST-PAPER STRUCTURE ADAPTATION *Inspired by the 2021 AA specimen Paper 2 finance/geometric-sequence structure.* An amount EUR P is invested on 1 January at a nominal annual rate of 5.8%, compounded quarterly. No further money is added or withdrawn. (a) Show that the year-end balances form a geometric sequence and find its annual common ratio to four significant figures. [4] (b) Find the first complete year in which the investment is more than twice its original value. [4] (c) If $P = 12500$, find the balance at the end of that year. [2] (d) Explain why using a common ratio of 1.058 would not be exact. [2]

29. PAST-PAPER STRUCTURE ADAPTATION *Inspired by the May 2023 AA SL Paper 2 financial comparison structure.* A car is worth EUR 46000 at time $t = 0$ and its value is modelled by

$$C(t) = 46000(0.92)^t,$$

where t is measured in years. At the same time, Alex invests EUR 18000 at 3.4% effective annual interest. (a) State the car's annual depreciation rate. [1] (b) Find the value of the car after 6 years. [2] (c) Find the value of Alex's investment after 6 years. [2] (d) Alex wants to buy the car at that time. Find the additional amount required. [2] (e) Find the first complete year for which Alex's investment is worth more than the car. [4]

30. PAST-PAPER STRUCTURE ADAPTATION *Inspired by the May 2024 AA HL Paper 2 special-first-year depreciation pattern.* A car is bought for EUR 48000. During the first year it loses 16% of its value. In every later year it loses 10% of the value at the start of that year. (a) Find its value at the end of year 1. [2] (b) Find its value after 11 complete years. [3] (c) Let V_n be its value after n years. Write a formula for V_n for $n \geq 1$. [2] (d) Find the first complete year for which V_n is below 15% of the original price. [4] (e) Explain why the exponent in your formula is $n - 1$, not n . [2]

31. PAST-PAPER STRUCTURE ADAPTATION *Inspired by the May 2025 AA HL Paper 2 depreciation and real-value pattern.* A piece of equipment costs EUR 42000 and depreciates at 14% per year. Separately, EUR 55000 is invested in an account that grows by 1.4% per month while inflation is 0.65% per month. (a) Find the equipment value after 9 years. [2] (b) Find the exact monthly multiplier for the real value of the investment. [2] (c) Hence find the first complete month in which the real value exceeds EUR 65000. [4] (d) A student instead subtracts the rates and uses a real growth rate of 0.75% per month. Explain why this is an approximation and comment on whether it changes the whole-month answer in this case. [4]

32. PAST-PAPER STRUCTURE ADAPTATION *Inspired by the November 2023 AA SL Paper 2 extended-response structure combining a growing payment sequence and compound investment.* Leila is offered two prize plans over 30 months. Plan A pays EUR 3200 at the beginning of every month. Plan B pays EUR 1400 in the first month and each later monthly payment is 5% greater than the previous payment. Omar invests EUR 90000 at 4.2% nominal annual interest compounded monthly. (a) Find the total amount Leila receives under Plan A. [2] (b) Find the total amount she receives under Plan B. [4] (c) Write an expression for the value of Omar's investment after m months. [2] (d) Leila chooses Plan B and keeps her payments as cash. Find the combined value of her received payments and Omar's investment at the end of month 8. [3] (e) Find the first complete month in which their combined value is at least EUR 175000. [4] (f) At the end of 30 months, Leila invests EUR 35000 for 5 years at a nominal annual rate of $r\%$, compounded quarterly. It grows to EUR 46000. Find r . [4]

Source note. The long-question structures draw on the public IB/IB Docs archive, including the 2021 AA specimen Paper 2 finance question, May 2023 AA SL Paper 2 finance question, November 2023 AA SL Paper 2 extended financial/geometric-series question, May 2024 AA HL Paper 2 depreciation question and May 2025 AA HL Paper 2 finance/real-value question. Official wording is not reproduced.