

CF MATHS

SAT Math Lecture Notes

Linear Equations and Systems

Build the algebra engine that powers a large part of SAT Math

■ Expanded theory edition

- complete linear-equation and systems theory
- Method A / Method B / Method C decision-making
- narrative explanations, traps, and SAT shortcuts
- graph interpretation and Desmos-style visual examples
- 15 selected official Question Bank paraphrases with detailed worked solutions
- separate 35-question worksheet with solutions collected only at the end

Independent CF Maths teaching material. Aligned to current College Board SAT Math skill descriptions. Official SAT question wording is not reproduced.

Expanded theory roadmap

How to read this new section

This chapter is not meant to feel like a list of formulas. Think of it as learning one language in several dialects. A line can appear as an equation, a graph, a table, a function, a word problem, or one equation inside a system. The SAT changes the disguise; the underlying mathematics stays the same.

For each idea, these notes deliberately give more than one route. **Method A** is usually the safest algebraic method. **Method B** is an equivalent shortcut or structural method. **Method C** is often a graph/Desmos route or a useful check. Your job is not to memorize three solutions to every problem. Your job is to recognize which route is shortest for the form in front of you.

1. The big story: what “linear” really means
2. Equality as a balance: the engine behind equation solving
3. Linear equations in one variable: Methods A, B, and C
4. Fractions, decimals, brackets, and nested structure
5. One solution, no solution, infinitely many solutions
6. Parameters: when the SAT asks for a constant, not for x
7. Linear functions and function notation
8. Slope: three ways to see the same rate
9. Forms of a line and when each form is useful
10. Tables, graphs, intercepts, and coordinate information
11. Parallel and perpendicular lines
12. Building linear models from words and units
13. Linear inequalities and boundary logic
14. Systems: substitution, elimination, graphing, Desmos
15. Special systems and parameter conditions
16. Systems in context: mixture, cost, tickets, rates
17. Desmos strategy without becoming calculator-dependent
18. Final SAT decision map and master trap list

What remains later in the chapter

After this expanded foundation, the original CF Maths worked-example sequence, 50 official Question Bank paraphrases/reconstructions, Desmos workshop, trap workshop, end-of-chapter practice, and full solutions continue unchanged. The purpose of the new material is to make those examples feel connected rather than isolated.

The big story: what “linear” really means

Imagine walking at a perfectly steady pace. Every extra minute adds the same distance. Imagine a taxi fare with a fixed starting charge and then the same amount added per kilometer. Imagine a tank that is filled by the same number of liters each minute. These are linear stories because the *change is constant*.

A linear relationship is not defined by the fact that its graph “looks like a line” only. That picture is a consequence of the deeper idea:

equal changes in x produce equal changes in y .

If x increases by 1 and y always increases by m , then the relationship has constant rate of change m .

The four faces of the same line

Suppose a service charges a fixed fee of 12 dollars and then 5 dollars per hour.

Words	Start at 12; add 5 for every extra hour.
Equation	$C = 5h + 12$.
Table	Each time h rises by 1, C rises by 5.
Graph	A straight line with slope 5 and vertical intercept 12.

The SAT often gives you one face and asks for another. Translation is therefore more important than memorization.

How to recognize linearity quickly

A relationship is linear if one of the following equivalent clues appears:

- an equation can be written as $y = mx + b$;
- the graph is a nonvertical straight line;
- the table has constant first differences in y for equal steps in x ;
- the context describes a constant rate plus, possibly, a fixed starting amount;
- in two variables, the equation can be written as $Ax + By = C$ with A and B not both zero.

Not every expression containing x is linear

x^2 , $\sqrt{x}$, $\frac{1}{x}$, 2^x , and $|x|$ are not linear over their full domains. Also, $xy = 6$ is not linear in x and y because the variables are multiplied together. The SAT may place these near linear-looking choices to test whether you are reading structure rather than keywords.

Independent and dependent variables

In $y = mx + b$, x is usually the input or independent variable and y is the output or dependent variable. In context, never assume the letters tell you which is which. Read the units.

If a problem says “the total cost C for n lessons”, then n is the input and C depends on n .

SAT decision. If you are lost in a word problem, write the units next to each variable before doing algebra. Units often reveal the slope immediately.

Equality as a balance: the engine behind solving equations

An equation says that two expressions have the same value. Think of a balanced scale. If you add 7 to the left side only, the scale is no longer balanced. If you add 7 to both sides, equality is preserved.

This gives the fundamental rule:

Do the same valid operation to both sides.

You may add, subtract, multiply, or divide both sides by the same nonzero number. You may also simplify each side using algebraic identities such as distribution and combining like terms.

Mini-story: why “move it across and change the sign” works

Solve $3x + 5 = 20$.

Students often say “move the 5 across and make it -5 ”. The real operation is

$$3x + 5 - 5 = 20 - 5,$$

so $3x = 15$ and $x = 5$. The shortcut phrase is fine only if you understand the balancing operation underneath it.

Inverse operations

Operations come in inverse pairs:

$$+a \leftrightarrow -a, \quad \times a \leftrightarrow \div a.$$

The goal is to undo operations around the variable in a sensible order.

Method A - Simplify first, then isolate

Use this when both sides contain brackets or like terms.

$$4(2x - 3) + 5 = 3x + 18$$

First simplify the left side:

$$8x - 12 + 5 = 3x + 18 \Rightarrow 8x - 7 = 3x + 18.$$

Now collect variable terms and constants:

$$5x = 25 \Rightarrow x = 5.$$

Method B - Undo outer structure before expanding

Sometimes expansion creates unnecessary work. For example,

$$5(2x - 1) = 35.$$

Divide by 5 first:

$$2x - 1 = 7 \Rightarrow 2x = 8 \Rightarrow x = 4.$$

This is faster than expanding to $10x - 5 = 35$, although both are correct.

Method C - Use equivalence or substitution as a check

If the answer choices are numerical, you can substitute a candidate value into the original equation. This is especially useful when the algebra is sign-heavy. It is a check, not usually the fastest first route.

The golden habit

Do not expand automatically. Before touching the algebra, ask: “Can I divide by a common factor first? Can I combine a repeated expression? Can I preserve useful structure?” Strong SAT algebra is often about doing *less*.

Linear equations in one variable: three routes

A linear equation in one variable can be rearranged to the form

$$ax + b = cx + d.$$

The solution is the value of x that makes the two sides equal.

Route 1: collect the x -terms

Example - variables on both sides

Solve $7x - 9 = 3x + 19$.

$$7x - 3x = 19 + 9 \Rightarrow 4x = 28 \Rightarrow \boxed{x = 7}.$$

Notice that we chose to keep the coefficient of x positive. You could move $7x$ to the right instead, but that creates $-4x = -28$. Both routes work; one is cleaner.

Route 2: exploit repeated expressions

Example - treat a repeated group as one object

Solve

$$4(3x + 2) - 5(3x + 2) = 9.$$

The repeated object is $(3x + 2)$. Factor it mentally:

$$(4 - 5)(3x + 2) = 9 \Rightarrow -(3x + 2) = 9.$$

So $3x + 2 = -9$, hence $3x = -11$ and

$$\boxed{x = -\frac{11}{3}}.$$

Expanding everything would still work, but structure gives the shorter route.

Route 3: solve a related expression instead of x

The SAT sometimes asks for an expression such as $2x + 5$ rather than for x itself.

Example - do not solve more than the question asks

If

$$6x + 15 = 51,$$

find $2x + 5$.

Because $6x + 15 = 3(2x + 5)$,

$$3(2x + 5) = 51 \Rightarrow \boxed{2x + 5 = 17}.$$

There is no need to find $x = 6$ first.

The SAT likes “hidden targets”

A common high-value habit is to circle the exact expression the question asks for. If the target already appears as a multiple or part of the given equation, manipulate the equation toward the target rather than automatically isolating x .

Decimal coefficients

Decimals are not mathematically different; they are just another notation for fractions. You can often multiply by a power of 10 to remove them.

For example,

$$0.3x + 1.2 = 2.7$$

can be multiplied by 10 to give

$$3x + 12 = 27,$$

so $x = 5$.

SAT decision. If the decimals are terminating and ugly, clearing them can reduce calculator dependence and sign errors.

Fractions, brackets, and nested structure

Fractions make many students feel that the equation has become harder. Usually it has not. The denominators are simply asking to be cleared.

Method A - Clear all denominators with the LCM

Solve

$$\frac{x-2}{3} + \frac{x+1}{4} = 5.$$

The least common multiple of 3 and 4 is 12. Multiply *every term* by 12:

$$4(x-2) + 3(x+1) = 60.$$

Then

$$4x - 8 + 3x + 3 = 60 \Rightarrow 7x = 65 \Rightarrow x = \frac{65}{7}.$$

Method B - Simplify a single fraction before clearing

If you see

$$\frac{6x+12}{3} = 10,$$

first simplify the numerator by factoring:

$$\frac{6(x+2)}{3} = 2(x+2) = 10.$$

So $x+2=5$ and $x=3$.

Method C - Cross-multiply only when there is one fraction on each side

For

$$\frac{2x-1}{5} = \frac{x+4}{3},$$

cross-multiplication is efficient:

$$3(2x-1) = 5(x+4).$$

But cross-multiplication is not a magic rule for sums of several fractions. For those, use the LCM method.

Distribution with a negative sign

The minus sign must multiply every term:

$$-2(3x-5) = -6x+10,$$

not $-6x-10$. A reliable mental sentence is: “negative two times three x , then negative two times negative five.”

Nested brackets

Work from the inside outward, but still look for structure.

$$3(2(x-1)+4) = 18.$$

Inside the bracket,

$$2(x-1)+4 = 2x-2+4 = 2x+2.$$

Then $3(2x+2) = 18$, so $2x+2 = 6$ and $x = 2$.

A useful checking habit

Substitute your answer into the *original* equation, not only your simplified version. This catches errors made during distribution or denominator clearing.

One solution, no solution, infinitely many solutions

Most equations have one solution, but the SAT loves the two special cases because they test whether you understand what an equation means.

Start with

$$ax + b = cx + d.$$

After collecting terms, three possibilities exist.

Case	What remains	Meaning
One solution	$(a - c)x = d - b$ with $a \neq c$	one value of x works
No solution	$0 = k$ where $k \neq 0$	impossible statement
Infinitely many	$0 = 0$	both sides were equivalent for every x

No solution

Solve

$$4(x + 2) = 4x + 11.$$

Expanding gives

$$4x + 8 = 4x + 11.$$

Subtract $4x$ from both sides:

$$8 = 11.$$

This is impossible, so there is no solution.

Infinitely many solutions

Solve

$$3(2x - 5) = 6x - 15.$$

Expanding gives

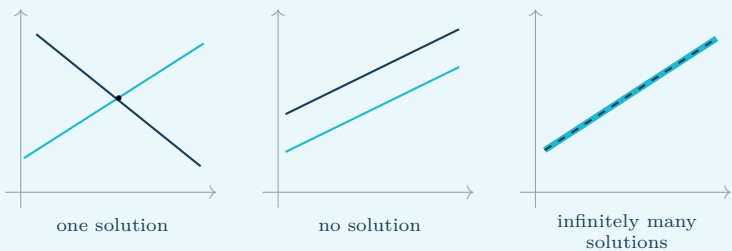
$$6x - 15 = 6x - 15.$$

Both sides are identical, so every real number works. There are infinitely many solutions.

Connection to graphs

Think of each side as a function of x .

- One solution: two lines meet once.
- No solution: two distinct parallel lines never meet.
- Infinitely many solutions: the two equations describe the same line.



This graph interpretation becomes extremely useful in parameter questions and systems.

Common trap. “No solution” does not mean “I could not solve it”. It is a mathematical conclusion, supported by an impossible equality such as $5 = 9$.

Parameters: when the question is really about the equation

A parameter is a constant whose value is not yet known. Typical letters are a , b , k , or p . The SAT may ask which value makes an equation have one solution, no solution, or infinitely many solutions.

Method A - Match coefficients after simplifying

For what value of k does

$$3(kx - 4) = 12x - 12$$

have infinitely many solutions?

Expand:

$$3kx - 12 = 12x - 12.$$

For the two sides to be identical for every x , the x -coefficients must match:

$$3k = 12 \Rightarrow \boxed{k = 4}.$$

Method B - Use the line picture

For what k does

$$4x + 8 = kx + 3$$

have no solution?

Each side is a line in x . No solution means same slope but different intercepts. Therefore

$$k = 4,$$

and the constants 8 and 3 are already different. Hence $\boxed{k = 4}$.

Method C - Use the coefficient test

For

$$ax + b = cx + d,$$

remember:

- if $a \neq c$, one solution;
- if $a = c$ and $b \neq d$, no solution;
- if $a = c$ and $b = d$, infinitely many solutions.

This compact test is one of the highest-value pieces of theory in the chapter.

The parameter is not the variable

If the question asks “for what value of k ”, do not automatically solve for x . The condition on k is often found by making coefficients match or differ in a particular way.

A harder pattern

Suppose

$$2(px + 5) = 6x + q.$$

For infinitely many solutions, expand first:

$$2px + 10 = 6x + q.$$

Then both coefficient and constant must match:

$$2p = 6, \quad q = 10.$$

So $p = 3$ and $q = 10$.

SAT decision. When two parameters appear, expect two matching conditions.

Linear functions and function notation

A function is a machine: one input goes in, exactly one output comes out. The notation

$$f(x) = 3x - 7$$

means “the output of function f when the input is x equals $3x - 7$.”

Read $f(x)$ as a sentence, not as multiplication

$f(4)$ does not mean $f \times 4$. It means “the value of the function when the input is 4.” So

$$f(4) = 3(4) - 7 = 5.$$

Similarly, $f(a) = 3a - 7$ and

$$f(a + 1) = 3(a + 1) - 7 = 3a - 4.$$

Method A - Direct substitution

If $f(x) = -5x + 2$, then

$$f(-3) = -5(-3) + 2 = 17.$$

Use brackets around negative inputs. This single habit prevents many sign errors.

Method B - A known input-output pair is a point

If $f(x) = mx + b$ and $f(4) = 11$, then $(4, 11)$ lies on the graph.

For example, if $f(x) = 2x + c$ and $f(4) = 11$,

$$11 = 2(4) + c \Rightarrow c = 3.$$

Method C - Use structure before evaluation

If

$$f(x) = 7x - 4$$

and you need $f(x + 2) - f(x)$, do not plug in a numerical value. Compute symbolically:

$$f(x + 2) = 7(x + 2) - 4 = 7x + 10,$$

so

$$f(x + 2) - f(x) = (7x + 10) - (7x - 4) = 14.$$

This tells us a deeper fact: increasing the input by 2 always increases the output by 14, because the slope is 7.

Linear function vocabulary

For $f(x) = mx + b$:

- m is the slope or rate of change;
- $b = f(0)$ is the initial value or y -intercept;
- the x -intercept solves $f(x) = 0$;
- a positive m means the function increases as x increases;
- a negative m means it decreases;
- $m = 0$ gives a horizontal line.

Slope: three ways to see the same rate

Slope measures vertical change per unit horizontal change:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

The order must be consistent: if the numerator is “second minus first”, the denominator must also be “second minus first”.

Method A - Read slope from $y = mx + b$

For

$$y = -\frac{3}{2}x + 7,$$

the slope is immediately

$$m = -\frac{3}{2}.$$

This means that for every increase of 2 in x , y decreases by 3.

Method B - Use two points

Through $(2, 5)$ and $(8, 17)$,

$$m = \frac{17 - 5}{8 - 2} = \frac{12}{6} = \boxed{2}.$$

You could reverse both differences:

$$\frac{5 - 17}{2 - 8} = \frac{-12}{-6} = 2.$$

The problem comes only when you reverse one difference but not the other.

Method C - Read constant change from a table

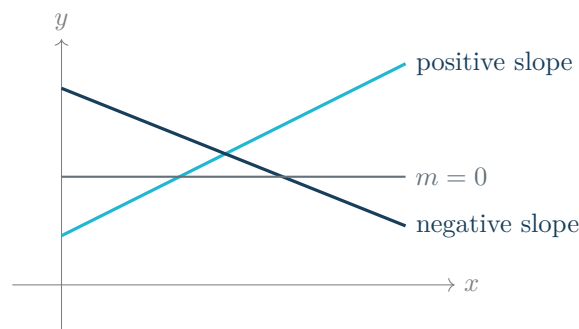
x	1	3	5	7
y	4	10	16	22

Each time x rises by 2, y rises by 6. Therefore

$$m = \frac{6}{2} = \boxed{3}.$$

Zero and undefined slope

A horizontal line has $\Delta y = 0$, so $m = 0$. A vertical line has $\Delta x = 0$, so the slope formula would divide by zero; its slope is undefined.



SAT decision. Slope is a *rate*. In context, attach units: dollars per hour, meters per second, points per game, and so on.

Forms of a line and when each form is useful

The same line can be written in several forms. The SAT may reward you for choosing the form that matches the information given.

Form A - Slope-intercept form

$$y = mx + b$$

Best when you need the slope m and vertical intercept b , or when graphing quickly.

Form B - Point-slope form

$$y - y_1 = m(x - x_1)$$

Best when you know a point (x_1, y_1) and the slope.

Example: slope 3 through $(4, -2)$:

$$y + 2 = 3(x - 4).$$

If needed, expand to $y = 3x - 14$.

Form C - Standard form

$$Ax + By = C$$

Best for elimination, intercepts, or when both variables appear naturally.

For $2x + 3y = 12$:

- set $y = 0$ to get the x -intercept $x = 6$;
- set $x = 0$ to get the y -intercept $y = 4$.

Converting forms

For

$$3x + 2y = 10,$$

solve for y :

$$2y = -3x + 10 \Rightarrow y = -\frac{3}{2}x + 5.$$

So the slope is $-\frac{3}{2}$ and the y -intercept is 5.

Method D - Two-point form by calculation

Given (x_1, y_1) and (x_2, y_2) :

1. calculate $m = \frac{y_2 - y_1}{x_2 - x_1}$;
2. use point-slope form with either point;
3. simplify only if the question requires another form.

Example - equation through two points

Find the line through $(1, 4)$ and $(5, 12)$.

$$m = \frac{12 - 4}{5 - 1} = 2.$$

Using $(1, 4)$:

$$y - 4 = 2(x - 1) \Rightarrow \boxed{y = 2x + 2}.$$

Common trap. Do not confuse the x -intercept with the y -intercept. At the x -intercept, $y = 0$. At the y -intercept, $x = 0$.

Tables, graphs, intercepts, and coordinate information

The SAT frequently tests the same line without ever writing $y = mx + b$. You may have to reconstruct the equation from a table, graph, or pair of points.

From a table

If the x -steps are equal, compare first differences in y .

x	-1	1	3	5
y	8	4	0	-4

Here x rises by 2 while y falls by 4, so

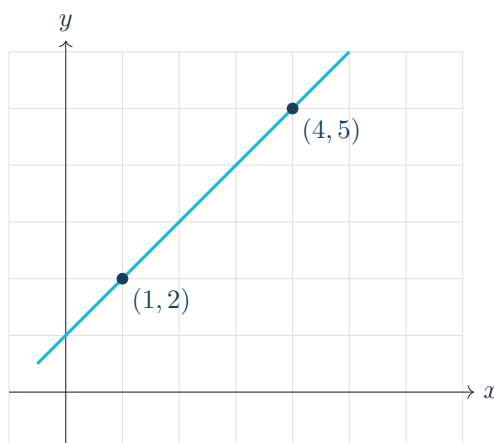
$$m = \frac{-4}{2} = -2.$$

Using point $(3, 0)$:

$$y - 0 = -2(x - 3) \Rightarrow y = -2x + 6.$$

From a graph

Choose two clear lattice points if possible. Avoid estimating from fuzzy points near the line when exact labeled points are available.



The slope is $(5 - 2)/(4 - 1) = 1$, so the line has form $y = x + b$. Using $(1, 2)$ gives $b = 1$, hence $y = x + 1$.

Intercepts carry meaning

In context, the y -intercept is often the value when the input is zero: starting balance, initial height, setup fee, initial temperature, and so on. The x -intercept is often when the output reaches zero: break-even time, depletion time, zero height, and similar events.

SAT decision. Ask whether an intercept makes sense in the context. A negative time may be algebraically correct but contextually irrelevant.

Parallel and perpendicular lines

Lines with the same slope never meet unless they are actually the same line. Therefore distinct parallel lines have equal slopes:

$$m_1 = m_2.$$

Perpendicular nonvertical lines have slopes that are negative reciprocals:

$$m_1 m_2 = -1.$$

So if one slope is $\frac{2}{3}$, the perpendicular slope is $-\frac{3}{2}$.

Method A - Read the slope, then reuse it

Find the line parallel to $y = -4x + 7$ through $(2, 5)$.

Parallel slope: $m = -4$.

$$y - 5 = -4(x - 2) \Rightarrow y = -4x + 13.$$

Method B - Negative reciprocal for perpendicularity

Find the line perpendicular to $y = \frac{1}{2}x - 3$ through $(4, 1)$.

Original slope = $\frac{1}{2}$, so perpendicular slope = -2 .

$$y - 1 = -2(x - 4) \Rightarrow y = -2x + 9.$$

Vertical and horizontal special case

The rule $m_1 m_2 = -1$ assumes both slopes are defined. A horizontal line $y = c$ is perpendicular to a vertical line $x = k$.

Using standard form without fully rearranging

For

$$Ax + By = C,$$

the slope is

$$-\frac{A}{B} \quad (B \neq 0).$$

For example, $3x - 2y = 8$ has slope $\frac{3}{2}$ because

$$-2y = -3x + 8 \Rightarrow y = \frac{3}{2}x - 4.$$

SAT decision. If the question asks only whether two lines are parallel, you may not need to find either intercept. Compare slopes and stop.

Building linear models from words and units

Word problems become easier when you stop hunting for a formula and instead ask two questions:

1. What is there at the beginning?
2. What is added or removed per unit?

This naturally creates

$$\text{output} = \text{rate} \times \text{input} + \text{initial value.}$$

Method A - Fixed amount + per-unit amount

A tutoring plan charges a 50 dollar registration fee plus 35 dollars per lesson. For n lessons,

$$C(n) = 35n + 50.$$

The fixed amount is the intercept; the per-lesson amount is the slope.

Method B - Start from two contextual points

A tank contains 140 liters after 2 minutes and 260 liters after 8 minutes. Assuming a linear model,

$$m = \frac{260 - 140}{8 - 2} = \frac{120}{6} = 20 \text{ L/min.}$$

Use $(2, 140)$:

$$V - 140 = 20(t - 2) \Rightarrow V = 20t + 100.$$

So the tank began with 100 liters.

Method C - Build from a table

If a table has constant first differences, divide change in output by change in input to get the rate. Then use any row to recover the initial value.

Units are part of the mathematics

If C is dollars and h is hours, the slope of C versus h has units dollars per hour. A slope answer with impossible units is a warning sign.

Decreasing model

A phone battery begins at 96% and decreases by 8 percentage points per hour. After t hours,

$$B(t) = 96 - 8t.$$

When will it reach 40%?

$$96 - 8t = 40 \Rightarrow 8t = 56 \Rightarrow \boxed{t = 7}.$$

Common trap. “Decreases by 8% each hour” can mean a percentage change, which is exponential. “Decreases by 8 percentage points each hour” is linear. Read the wording carefully.

Linear inequalities: equations with a region of answers

An equation usually asks for exact equality. An inequality describes a range:

$$x < 4, \quad x \geq -2, \quad 3x + 5 > 17.$$

Solve inequalities almost like equations, with one crucial exception.

Why the sign reverses

If $2 < 5$, multiplying both sides by -1 gives $-2 > -5$. Negative multiplication reverses order on the number line. Therefore whenever you multiply or divide an inequality by a negative number, reverse the inequality sign.

Method A - Ordinary isolation

Solve

$$5x - 7 \leq 18.$$

Then

$$5x \leq 25 \Rightarrow \boxed{x \leq 5}.$$

Method B - Negative coefficient, so flip the sign

Solve

$$-3x + 4 > 19.$$

Then

$$-3x > 15.$$

Divide by -3 and reverse the sign:

$$\boxed{x < -5}.$$

Context: integer restrictions

A problem may produce

$$n > 9.2,$$

but if n is the number of whole tickets, people, or months, the smallest possible integer is 10.

Minimum number in context

A student has 35 points and earns 18 points per completed task. How many whole tasks are needed to exceed 200 points?

$$35 + 18t > 200 \Rightarrow 18t > 165 \Rightarrow t > 9.166\dots$$

Since t is a whole number, the answer is $\boxed{10}$.

Two-variable inequalities

An inequality such as

$$y \leq 2x + 3$$

describes a region. The boundary is $y = 2x + 3$. Because equality is included, the boundary line is solid. For $<$ or $>$, the boundary is not included.

SAT decision. In SAT multiple-choice graph questions, test a simple point such as $(0,0)$ if it is not on the boundary. Substitute it to determine which side should be shaded.

Systems of equations: one point satisfying two stories

A system asks for values that satisfy two equations at the same time. Geometrically, the solution is the intersection point of two lines.

Method A - Substitution

Best when one variable is already isolated or has coefficient 1.

Solve

$$\begin{cases} y = 2x + 1, \\ 3x + y = 16. \end{cases}$$

Substitute $2x + 1$ for y :

$$3x + (2x + 1) = 16 \Rightarrow 5x = 15 \Rightarrow x = 3.$$

Then $y = 2(3) + 1 = 7$. So

$$(x, y) = (3, 7).$$

Method B - Elimination

Best when coefficients already match or can be made to match cheaply.

Solve

$$\begin{cases} 2x + 3y = 19, \\ 2x - y = 7. \end{cases}$$

Subtract the second equation from the first:

$$4y = 12 \Rightarrow y = 3.$$

Then $2x - 3 = 7$, so $x = 5$. Hence $(5, 3)$.

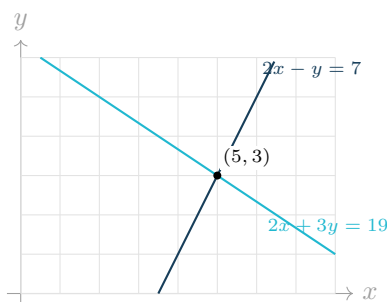
Method C - Graph / Desmos

Enter both equations. The intersection gives the solution. This is excellent when the arithmetic is ugly or when the question is really about the intersection. It is less useful when the answer must be an exact symbolic expression that is easy to derive algebraically.

For the system

$$\begin{cases} 2x + 3y = 19, \\ 2x - y = 7, \end{cases}$$

Desmos shows the two lines meeting at $(5, 3)$.



Choosing between substitution and elimination

Situation	Good first choice
$y = \text{something}$	substitution
Opposite coefficients already present	elimination by adding
Matching coefficients already present	elimination by subtracting
Messy decimals / graphing question	Desmos or graphing
Exact parameter condition	algebra first

Common trap. After finding one variable, substitute back into an *original* equation. Do not stop with only x when the system asks for an ordered pair or for an expression involving both variables.

Special systems and parameter conditions

Two nonvertical lines can intersect once, never intersect, or be the same line. That gives the three system outcomes.

Outcome	Slope/intercept picture	Number of solutions
Different slopes	lines meet once	one
Same slope, different intercepts	parallel distinct lines	none
Same slope, same intercept	same line	infinitely many

Parameter for no solution

For what value of k does the system

$$\begin{cases} y = 4x + 2, \\ y = kx - 5 \end{cases}$$

have no solution?

No solution means equal slopes and different intercepts. Therefore

$$\boxed{k = 4}.$$

The intercepts 2 and -5 are already different.

Parameter for infinitely many solutions

For what values of a and b does

$$\begin{cases} 2x + 4y = 10, \\ ax + by = 5 \end{cases}$$

have infinitely many solutions?

The second equation must be exactly one half of the first:

$$a = 1, \quad b = 2.$$

Thus $\boxed{a = 1, b = 2}$.

Ratio method in standard form

For

$$A_1x + B_1y = C_1, \quad A_2x + B_2y = C_2,$$

identical lines require all three coefficients to be proportional by the same factor. Parallel distinct lines have proportional x and y coefficients but a different constant ratio.

SAT decision. In a parameter system, ask first whether the question is about the *solution point* or about the *number of solutions*. Those are different tasks.

Systems in context: translate before you solve

Many system problems are really two unknown quantities connected by two facts. The hardest part is usually writing the equations correctly.

Method A - Define variables, then translate sentence by sentence

Adult tickets cost 12 dollars and student tickets cost 8 dollars. A total of 50 tickets were sold for 520 dollars. Let a be adult tickets and s be student tickets.

$$\begin{cases} a + s = 50, \\ 12a + 8s = 520. \end{cases}$$

Multiply the first equation by 8:

$$8a + 8s = 400.$$

Subtract from the revenue equation:

$$4a = 120 \Rightarrow a = 30,$$

so $s = 20$.

Method B - Use a “total” equation to substitute immediately

From $a + s = 50$, write $s = 50 - a$. Substitute into

$$12a + 8s = 520.$$

Then

$$12a + 8(50 - a) = 520 \Rightarrow 4a = 120 \Rightarrow a = 30.$$

This is the same mathematics as elimination; choose the form you find cleaner.

Common system story types

- two ticket or item types: total number + total value;
- mixture problems: total quantity + total amount of a component;
- two rates or plans: equal total cost at a break-even point;
- age/count problems: one relationship now + another relationship later;
- geometry: perimeter/angle relationships giving two equations;
- coins: total number + total monetary value.

Units are your built-in error detector

In the ticket example, $a + s$ has units of tickets, so it can equal 50 tickets. The expression $12a + 8s$ has units of dollars, so it can equal 520 dollars. If you accidentally write $12a + 8s = 50$, the units expose the mistake immediately.

Desmos strategy without becoming calculator-dependent

Desmos is available in the digital SAT, and it is powerful. The goal is not to graph everything. The goal is to use Desmos where it reduces time or error.

Desmos Route A - Intersection of two expressions

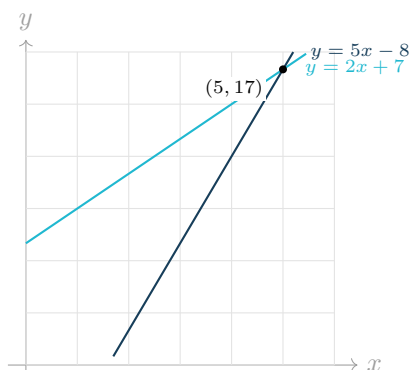
To solve

$$2x + 7 = 5x - 8,$$

you can enter

$$y = 2x + 7, \quad y = 5x - 8.$$

The x -coordinate of the intersection is the solution. Algebra is faster here, but the graph is a good check.

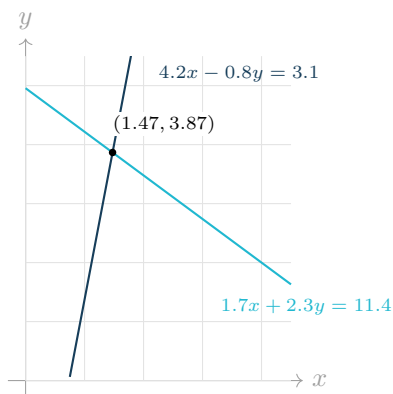


Desmos Route B - Systems with ugly coefficients

For a system such as

$$\begin{cases} 1.7x + 2.3y = 11.4, \\ 4.2x - 0.8y = 3.1, \end{cases}$$

Desmos can be efficient. Enter both equations directly and inspect the intersection.



Desmos Route C - Parameter experiments

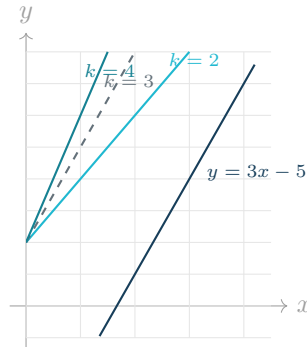
If you are unsure what “no solution” looks like, graph a family such as

$$y = kx + 2$$

against

$$y = 3x - 5$$

and vary k . You will see that $k = 3$ makes the lines parallel. This is useful for understanding, but on test day the slope logic is faster.



When algebra is usually faster

- simple one-variable equations;
- fraction equations with a clear LCM;
- coefficient-matching parameter questions;
- exact expressions where the requested form is symbolic;
- systems that eliminate in one line.

When Desmos is often faster

- messy decimals;
- graph/intersection questions;
- checking a model against a visual;
- quickly confirming the number of intersections;
- problems where answer choices are approximate numerical values.

Calculator discipline

Desmos can confirm a wrong equation perfectly. The translation step is still yours. A graphing tool does not protect you from defining the model incorrectly.

Final SAT decision map and master trap list

When a linear problem appears, do not start calculating immediately. Run this short mental checklist.

The 10-second decision tree

1. **What form am I looking at?** Equation, function, graph, table, context, inequality, or system?
2. **What exactly is being asked?** x , an expression in x , a slope, an intercept, a parameter, or a number of solutions?
3. **Is useful structure already present?** Common factor, repeated expression, matching coefficients, clear slope, fixed fee + rate?
4. **Can I do less algebra?** Divide first, clear denominators once, eliminate directly, or match coefficients.
5. **Would Desmos genuinely save time?** Use it when it reduces work, not as a reflex.

Master trap list

1. **Expanding too early.** Preserve structure when division or factoring is easier.
2. **Changing only one side.** Equation operations must preserve equality.
3. **Forgetting a denominator term.** When clearing fractions, multiply every term.
4. **Sign errors in distribution.** A negative multiplier changes every sign inside the bracket.
5. **Cross-multiplying a sum of fractions.** Use the LCM unless there is one fraction on each side.
6. **Solving for the wrong target.** If the question asks for $2x + 5$, do not automatically solve for x first.
7. **Confusing function notation with multiplication.** $f(3)$ means the output at input 3.
8. **Mixing slope difference order.** Keep numerator and denominator subtraction order consistent.
9. **Confusing intercepts.** At the x -intercept, $y = 0$; at the y -intercept, $x = 0$.
10. **Forgetting the inequality flip.** Reverse the sign only when multiplying or dividing by a negative number.
11. **Ignoring integer context.** A result like $n > 9.2$ may mean at least 10 whole items.
12. **Mixing parallel and perpendicular rules.** Parallel: same slope. Perpendicular: negative reciprocal.
13. **Stopping after one variable in a system.** Substitute back if the question needs the ordered pair or another expression.
14. **Treating no solution as a failure.** It is a valid outcome indicated by a contradiction.
15. **Using Desmos before modeling.** The calculator solves the equation you enter, not the story you intended.

Master mixed example - choose the method before computing

A gym charges a fixed joining fee and then a constant monthly fee. After 4 months the total paid is 178 dollars; after 10 months it is 340 dollars.

Step 1: identify the structure. This is linear: total cost = monthly rate $\times$ months + joining fee.

Method A - slope from two points.

$$m = \frac{340 - 178}{10 - 4} = \frac{162}{6} = 27.$$

So the monthly fee is 27 dollars. Use (4, 178):

$$178 = 27(4) + b \Rightarrow b = 70.$$

Thus

$$C(n) = 27n + 70.$$

Method B - system.

$$\begin{cases} 4m + b = 178, \\ 10m + b = 340. \end{cases}$$

Subtract to get $6m = 162$, so $m = 27$, then $b = 70$.

Method C - Desmos. Plot the points (4, 178) and (10, 340) and use a linear model or solve the two equations. The answer is the same; the best SAT route here is usually Method A because the rate is visible immediately from the two points.

The chapter in one sentence

Linear algebra on the SAT is the art of recognizing a constant rate, translating between representations, and choosing the shortest valid route while keeping the meaning of the quantities visible.

15 exam-style worked questions

How to use this final workshop

The theory is now complete. These 15 questions are deliberately selected to cover different SAT patterns rather than repeat the same algebraic move. Before reading each solution, decide your method first: direct algebra, structure, rate reasoning, intercept logic, or a graph/Desmos check. The source IDs are included for traceability to the official SAT Math Question Bank; all wording and numerical values shown here are paraphrased/reconstructed CF Maths teaching material.

Exam Question 1 - Infinite solutions by matching coefficients

Source ID: ac472881

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. In the equation

$$4(px - 9) = 16x - 36,$$

p is a constant. For what value of p does the equation have infinitely many solutions?

Difficulty: Hard

Detailed solution

Expand the left side:

$$4px - 36 = 16x - 36.$$

For infinitely many solutions, both sides must be identical for every x . The constants already match, so the coefficients of x must match:

$$4p = 16 \quad \Rightarrow \quad \boxed{p = 4}.$$

Quick SAT route. For infinitely many solutions, make the two expressions identical.

Common trap. Trying to solve for x before enforcing the identity condition.

Exam Question 2 - No solution by matching the variable coefficient

Source ID: 12ee1edc

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. In the equation

$$4(kx + 2) = 16x + 13,$$

what value of k makes the equation have no solution?

Difficulty: Medium

Detailed solution

Expand:

$$4kx + 8 = 16x + 13.$$

No solution occurs when the x -coefficients match but the constants do not. Thus

$$4k = 16 \quad \Rightarrow \quad \boxed{k = 4}.$$

Then the equation becomes $16x + 8 = 16x + 13$, which is impossible.

Quick SAT route. Match the variable coefficients, then check that the constants are different.

Common trap. Matching both coefficient and constant; that would create infinitely many solutions.

Exam Question 3 - Equivalent linear equations

Source ID: e7b6f0d1

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. Which equation has the same solution as $4x + 3 = 3$?

Difficulty: Easy

A. $5x + 3 = 3$

B. $8x + 6 = 6$

C. $4x = 3$

D. $4x + 5 = 3$

Detailed solution

The original equation gives $4x = 0$, so $x = 0$. Multiplying *every term* of the original equation by 2 gives

$$8x + 6 = 6,$$

which has the same solution. Hence **B**.

Quick SAT route. Equivalent equations come from performing the same valid operation on both sides.

Common trap. Changing only one term or one side.

Exam Question 4 - Constant-rate context

Source ID: 2937ef4f

Difficulty: Hard

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A storage tank initially contains 17,000 bushels of grain. Material is removed at a constant rate. After 4 hours, 12,200 bushels remain. If the same rate continues, after how many total hours will 2,600 bushels remain?

Detailed solution

In 4 hours, the amount removed is

$$17,000 - 12,200 = 4,800,$$

so the rate is $4,800/4 = 1,200$ bushels per hour. Let t be the total number of hours:

$$17,000 - 1,200t = 2,600.$$

Thus $1,200t = 14,400$, giving **$t = 12$** .

Quick SAT route. Find the rate first, then write initial amount $-$ rate $\times$ time $=$ target amount.

Common trap. Using the amount remaining as though it were the amount removed.

Exam Question 5 - Fixed fee plus hourly rate

Source ID: 76f29fa5

Difficulty: Medium

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A rental company charges a fixed fee of \$100 plus \$60 per hour. A customer pays \$580 in total. For how many hours was the item rented?

Detailed solution

Let h be the number of hours. Then

$$100 + 60h = 580.$$

So $60h = 480$ and therefore **$h = 8$** .

Quick SAT route. Use total $=$ fixed fee $+ \text{rate} \times \text{quantity}$.

Common trap. Dividing the full total by the hourly rate without removing the fixed fee first.

Exam Question 6 - Translate words into an equation

Source ID: 4de87c9a

Difficulty: Easy

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. "13 more than 8 times a number x is 39." Which equation represents the statement?

A. $8x + 13 = 39$

B. $8(x + 13) = 39$

C. $8x - 13 = 39$

D. $x + 104 = 39$

Detailed solution

“8 times a number” gives $8x$. “13 more than” means add 13. Therefore

$$8x + 13 = 39,$$

so A.

Quick SAT route. Translate each phrase separately before combining them.

Common trap. Placing the addition inside the multiplication.

Exam Question 7 - Solve first, then evaluate

Source ID: 7a5a74a6

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. If

Difficulty: Medium

$$8x - 7 = 9,$$

what is the value of $5x + 7$?

Detailed solution

First solve for x :

$$8x = 16 \Rightarrow x = 2.$$

Now evaluate the requested expression:

$$5x + 7 = 5(2) + 7 = \boxed{17}.$$

Quick SAT route. Do not stop after finding x ; answer what the question actually asks.

Common trap. Reporting $x = 2$ instead of the value of $5x + 7$.

Exam Question 8 - Recover a linear function from values

Source ID: e9908930

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A linear function f satisfies

Difficulty: Hard

$$f(0) = -2, \quad f(4) = -6, \quad f(7) = -9.$$

What is $f(11)$?

Detailed solution

Use two points to find the slope:

$$m = \frac{-6 - (-2)}{4 - 0} = -1.$$

Since $f(0) = -2$, the intercept is -2 , so

$$f(x) = -x - 2.$$

Hence

$$f(11) = -11 - 2 = \boxed{-13}.$$

Quick SAT route. For a linear function, any two input-output pairs determine the constant rate.

Common trap. Using the change in outputs without dividing by the change in inputs.

Exam Question 9 - Use both intercepts

Source ID: 620fe971

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. The graph of

Difficulty: Medium

$$h(x) = -8x + 56$$

has x -intercept $(a, 0)$ and y -intercept $(0, b)$. What is $a + b$?

Detailed solution

For the x -intercept, set $h(x) = 0$:

$$0 = -8x + 56 \Rightarrow x = 7,$$

so $a = 7$. For the y -intercept, set $x = 0$, giving $b = 56$. Therefore

$$a + b = 7 + 56 = \boxed{63}.$$

Quick SAT route. x -intercept: output is 0. y -intercept: input is 0.

Common trap. Reversing the two intercept conditions.

Exam Question 10 - Build and evaluate a linear model

Source ID: 1a2a0f59

Difficulty: Hard

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A storage service has a fixed starting charge of \$50 and then costs \$5 per unit. Let $C(n)$ be the total cost for n units. Write $C(n)$ and find $C(15)$.

Detailed solution

The fixed charge is the intercept and the per-unit charge is the slope:

$$C(n) = 5n + 50.$$

Then

$$C(15) = 5(15) + 50 = \boxed{125}.$$

Quick SAT route. Fixed amount $\rightarrow$ constant term; per-unit amount $\rightarrow$ coefficient.

Common trap. Multiplying the fixed fee by the number of units.

Exam Question 11 - A second model in a different context

Source ID: 930c2990

Difficulty: Easy

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A delivery service has a fixed starting charge of \$30 and then costs \$25 per unit. Let $C(n)$ be the total cost for n units. Write the model and find $C(11)$.

Detailed solution

$$C(n) = 25n + 30,$$

so

$$C(11) = 25(11) + 30 = 275 + 30 = \boxed{305}.$$

Quick SAT route. Read the slope and intercept directly from the language of the model.

Common trap. Confusing the fixed charge with the per-unit rate.

Exam Question 12 - Read a line from slope and intercept

Source ID: 415ab1d2

Difficulty: Hard

Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced. A linear function has slope -5 and y -intercept $(0, 5)$. What is the y -coordinate when $x = -2$?

Detailed solution

The equation is

$$y = -5x + 5.$$

At $x = -2$,

$$y = -5(-2) + 5 = 10 + 5 = \boxed{15}.$$

Quick SAT route. Write $y = mx + b$ immediately from the slope and y -intercept.

Common trap. Using the x -intercept as the constant term.

Exam Question 13 - Find a missing constant**Source ID:** 00723d16**Difficulty:** Hard*Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced.* For the linear function

$$f(x) = x + c,$$

it is known that $f(8) = 15$. What is c ?**Detailed solution**

Substitute the known input-output pair:

$$15 = 8 + c.$$

Therefore $\boxed{c = 7}$.**Quick SAT route.** A known value such as $f(8) = 15$ gives the point $(8, 15)$. Substitute it.**Common trap.** Treating the output 15 as a slope.**Exam Question 14 - Missing constant with negative slope****Source ID:** 98d3393a**Difficulty:** Hard*Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced.* For the linear function

$$f(x) = -3x + c,$$

it is known that $f(7) = -17$. What is c ?**Detailed solution**

$$-17 = -3(7) + c = -21 + c.$$

Hence $\boxed{c = 4}$.**Quick SAT route.** Substitute the point first; then isolate the constant.**Common trap.** Losing the negative sign in $-3(7)$.**Exam Question 15 - Missing constant under time pressure****Source ID:** 38a43902**Difficulty:** Medium*Official College Board SAT Math Question Bank item - CF Maths paraphrase/reconstruction. Official wording is not reproduced.* For the linear function

$$f(x) = 6x + c,$$

it is known that $f(5) = 21$. What is c ?**Detailed solution**

$$21 = 6(5) + c = 30 + c,$$

so $\boxed{c = -9}$.**Quick SAT route.** Think “output minus mx ”; here $c = 21 - 30 = -9$.**Common trap.** Assuming the constant must be positive because the output is positive.**Where to go next**

The separate Chapter 1 worksheet contains the remaining 35 paraphrased Question Bank items with no solutions shown beside the questions. Complete the entire worksheet first; the worked solutions are collected only at the very end.

Additional SAT-Style Multiple Choice

Chapter 1: Linear Equations and Systems

These questions are new CF Maths questions built from common Digital SAT patterns: equation structure, systems, line models, parameters, slope, intercepts, and context. Try them without looking at the answer key. Several are deliberately designed so that recognizing the structure is faster than carrying out long algebra.

1. Solve $5(2x - 3) - 4 = 3(x + 7)$.
 - A. $\frac{20}{7}$
 - B. $\frac{40}{7}$
 - C. 8
 - D. 10
2. Solve $\frac{x-4}{3} + \frac{x+2}{6} = 5$.
 - A. 8
 - B. 10
 - C. 12
 - D. 14
3. For what value of k does $4x + 7 = kx - 5$ have no solution?
 - A. -4
 - B. 0
 - C. 4
 - D. 12
4. For what value of a does $3(ax - 2) = 12x - 6$ have infinitely many solutions?
 - A. 1
 - B. 2
 - C. 3
 - D. 4
5. Solve the system $2x + 3y = 19$ and $2x - y = 7$.
 - A. (4, 3)
 - B. (5, 3)
 - C. (5, 2)
 - D. (6, 1)
6. Solve the system $y = 3x - 4$ and $2x + y = 16$.
 - A. (2, 2)
 - B. (3, 5)
 - C. (4, 8)
 - D. (5, 11)
7. Adult tickets cost \$14 and student tickets cost \$9. A total of 50 tickets brings in \$570. How many adult tickets were sold?
 - A. 20

- B. 24
C. 26
D. 30
8. A line passes through $(2, 5)$ and $(8, 23)$. What is its slope?
A. 2
B. 3
C. 4
D. 6
9. Which equation represents the line through $(4, -1)$ parallel to $y = 3x + 7$?
A. $y = 3x - 13$
B. $y = -3x + 11$
C. $y = \frac{1}{3}x - \frac{7}{3}$
D. $y = -\frac{1}{3}x + \frac{1}{3}$
10. What is the slope of a line perpendicular to $4x - 2y = 9$?
A. -2
B. $-\frac{1}{2}$
C. $\frac{1}{2}$
D. 2
11. A taxi fare is modeled by $C = 6.50 + 2.40m$, where m is miles traveled. What does 2.40 represent?
A. The starting fare
B. The total fare after 1 mile
C. The cost per mile
D. The number of miles in a trip
12. The amount of water in a tank is $W = 180 - 12t$, where t is hours. What is the t -intercept of the graph?
A. 12
B. 15
C. 168
D. 180
13. A linear function has values $f(1) = 7$ and $f(5) = 19$. Which equation defines f ?
A. $f(x) = 2x + 5$
B. $f(x) = 3x + 4$
C. $f(x) = 4x + 3$
D. $f(x) = 5x + 2$
14. If $f(x) = 7x - 11$, for what value of x is $f(x) = 45$?
A. 6
B. 7
C. 8
D. 9

15. Plan A costs $20 + 4x$ dollars and Plan B costs $44 + 2x$ dollars for x units of use. For what value of x do the plans cost the same?
- A. 6
 - B. 10
 - C. 12
 - D. 16
16. The line $y = mx + 7$ passes through $(6, 31)$. What is m ?
- A. 3
 - B. 4
 - C. 5
 - D. 6
17. For what value of k does the system $3x + ky = 8$ and $6x + 10y = 21$ have no solution?
- A. 2
 - B. 5
 - C. 10
 - D. 20
18. Which statement describes the graphs of $y = 2x - 5$ and $4x - 2y = 10$?
- A. They intersect once
 - B. They are distinct parallel lines
 - C. They are the same line
 - D. They are perpendicular
19. The formula $P = 18 + 5n$ gives the perimeter of a design. Which expression gives n in terms of P ?
- A. $\frac{P-18}{5}$
 - B. $\frac{P+18}{5}$
 - C. $5P - 18$
 - D. $\frac{18-P}{5}$
20. A linear model passes through $(10, 72)$ and $(25, 117)$. What is the predicted value when $x = 18$?
- A. 90
 - B. 96
 - C. 102
 - D. 108
21. A repair company charges a fixed fee plus \$65 per hour. A 4-hour job costs \$335. What is the fixed fee?
- A. \$65
 - B. \$75
 - C. \$95
 - D. \$260
22. If $g(x) = \frac{3x+5}{2}$, for what value of x is $g(x) = 19$?
- A. 9
 - B. 10

C. 11

D. 12

23. What is the x -intercept of $5x + 2y = 30$?

A. $(0, 15)$

B. $(5, 0)$

C. $(6, 0)$

D. $(15, 0)$

24. A company has profit $P = 18n - 270$. What is the smallest whole number of items n for which the profit is positive?

A. 15

B. 16

C. 17

D. 18

Answer Key

Use this only after completing the set. If you miss a question, identify the idea first: equation balance, line structure, system structure, parameter condition, or context interpretation.

- | | | | |
|------|-------|-------|-------|
| 1. B | 7. B | 13. B | 19. A |
| 2. C | 8. B | 14. C | 20. B |
| 3. C | 9. A | 15. C | 21. B |
| 4. D | 10. B | 16. B | 22. C |
| 5. B | 11. C | 17. B | 23. C |
| 6. C | 12. B | 18. C | 24. B |