

CF MATHS

SAT Math Lecture Notes

Ratios, Percentages and Proportions

Turn comparison problems into unit rates, multipliers, scale factors, and clean equations

■ Expanded theory edition

- complete ratio, rate, proportion, unit, and percentage theory
- Method A / Method B / Method C decision-making throughout
- narrative explanations, visual models, traps, and SAT shortcuts
- dimensional analysis and unit-cancellation diagrams
- proportional graphs and Desmos-style visual examples
- 15 selected Educator Question Bank inspired paraphrases with detailed solutions
- separate 35-question worksheet with all solutions collected only at the end

Independent CF Maths teaching material. Aligned to SAT Math Problem-Solving and Data Analysis skill patterns in the supplied Educator Question Bank export. Official question wording is not reproduced; worked exam examples use changed values, contexts, and wording.

Chapter roadmap

The big story

This chapter is about *comparison*. A ratio compares two quantities. A rate compares quantities with different units. A proportion says that two ratios describe the same relationship. A percentage compares a part to a whole out of 100. A scale factor tells you how one version of an object relates to another. The SAT changes the story, but the mathematical engine is usually one of these few ideas.

The most important habit is to ask, before doing arithmetic: *What is being compared to what, and what are the units?*

1. Ratios and order
2. Equivalent ratios
3. Ratio bars and splitting totals
4. Rates and unit rates
5. Compound units
6. Dimensional analysis
7. Multi-stage unit conversions
8. Proportions: three methods
9. Direct proportion $y = kx$
10. Tables and proportional graphs
11. Proportional vs merely linear
12. Percent means “per 100”
13. Part, whole, percent
14. Percent multipliers
15. Percent increase and decrease
16. Reverse percentages
17. Repeated percentage changes
18. Tax, tip, discount, markup
19. Percentage points vs percent change
20. Nested and chained percentages
21. Density and derived rates
22. Scale drawings and scale factors
23. Area and volume scale factors
24. Sample estimates
25. Desmos strategy
26. Master decision map
27. 15 exam-style worked examples

Ratios: order matters

Suppose a basket contains 6 red balls and 10 blue balls. The ratio of red to blue is

$$6 : 10 = 3 : 5.$$

The ratio of blue to red is

$$10 : 6 = 5 : 3.$$

These are not the same statement. Ratio questions are often easy mathematically and difficult linguistically because one reversed word can reverse the answer.

Think of a ratio as a sentence

“Red to blue is 3 : 5” means that for every 3 equal parts assigned to red, 5 equal parts are assigned to blue. The numbers do not have to be the actual counts. They describe the relationship.



Method A - Simplify the ratio

If a class has 18 left-handed students and 30 right-handed students, then

$$18 : 30 = \frac{18}{30} = \frac{3}{5} = 3 : 5.$$

Divide both parts by the same common factor.

Method B - Keep the order from the wording

If the question asks for the ratio of right-handed to left-handed students, write

$$30 : 18 = 5 : 3.$$

Write the labels above the numbers before simplifying if the wording is long.

Common trap. A ratio 3 : 5 does not mean 3 out of 5. It means 3 compared with 5. If the two groups make the whole, then the first group is $3/(3 + 5) = 3/8$ of the total.

Equivalent ratios: same relationship, different scale

Multiplying or dividing both parts of a ratio by the same positive number produces an equivalent ratio:

$$3 : 5 = 6 : 10 = 12 : 20 = 30 : 50.$$

The multiplier is a **scale factor**.

A quick scale-factor example

The ratio of flour to sugar is 4 : 3. If 20 cups of flour are used, the multiplier is

$$20 \div 4 = 5.$$

So the sugar amount is

$$3(5) = 15 \text{ cups.}$$

Method A - Scale factor

Use this when one number is obviously a multiple of the original ratio part.

$$4 : 7 = 20 : x \quad \Rightarrow \quad \times 5 \quad \Rightarrow \quad x = 35.$$

Method B - Unit ratio

Convert the relationship to “per 1.” If 4 parts correspond to 20, then 1 part corresponds to 5. Seven parts correspond to $7(5) = 35$.

Method C - Proportion equation

Write

$$\frac{4}{7} = \frac{20}{x}$$

and cross multiply:

$$4x = 140 \Rightarrow x = 35.$$

This always works when the ratio order is correct, but it can be slower than seeing the scale factor directly.

Splitting a total in a ratio

If a total is split in the ratio $a : b$, the total number of equal parts is $a + b$.

Split 96 in the ratio 5 : 3

There are $5 + 3 = 8$ equal parts. Each part is

$$96 \div 8 = 12.$$

So the two shares are

$$5(12) = 60, \quad 3(12) = 36.$$

The ratio-bar picture

A ratio bar is often safer than jumping directly to a formula. It makes the whole visible.



SAT decision. If the total and ratio are given, add the ratio numbers first. If one part and the ratio are given, find the scale factor first.

Rates and unit rates

A rate compares quantities measured in different units:

$$\frac{420 \text{ miles}}{7 \text{ hours}} = 60 \text{ miles per hour.}$$

A **unit rate** has a denominator of 1 unit.

Read the word “per” as division

Dollars per pound means

$$\frac{\text{dollars}}{\text{pounds}}.$$

Miles per hour means

$$\frac{\text{miles}}{\text{hours}}.$$

People per square mile means

$$\frac{\text{people}}{\text{square miles}}.$$

The units tell you what should be divided by what.

Method A - Divide to find the unit rate

A machine produces 168 components in 6 hours. Its rate is

$$168 \div 6 = 28 \text{ components/hour.}$$

Method B - Build from the unit rate

At 28 components per hour, in 9 hours the machine produces

$$28(9) = 252 \text{ components.}$$

Method C - Proportion

$$\frac{168}{6} = \frac{x}{9} \Rightarrow 6x = 1512 \Rightarrow x = 252.$$

Use this when the relationship is easier to see as two equivalent rates.

Compound units and dimensional analysis

Some SAT rates contain more than one unit: dollars per cubic inch, people per square kilometer, miles per second, pounds per square inch, and so on. The safest method is to treat units like algebraic factors.

Units can cancel

Suppose a satellite travels at 5.2 miles per second. To convert to miles per hour,

$$5.2 \frac{\text{mi}}{\text{s}} \times 60 \frac{\text{s}}{\text{min}} \times 60 \frac{\text{min}}{\text{h}}.$$

Seconds cancel. Minutes cancel. The only remaining unit is miles per hour.

$$5.2 \frac{\text{mi}}{\text{s}} \times 60 \frac{\text{s}}{\text{min}} \times 60 \frac{\text{min}}{\text{h}} = 18,720 \frac{\text{mi}}{\text{h}}$$

Cancellation check

If the unwanted unit does not cancel, flip the conversion factor. Do not memorize whether to multiply or divide; choose the fraction orientation that cancels the old unit.

Multi-stage unit conversions

The SAT may require more than one conversion.

Quarts to fluid ounces

Use

$$1 \text{ quart} = 4 \text{ cups}, \quad 1 \text{ cup} = 8 \text{ fl oz.}$$

Then

$$19 \text{ qt} \times \frac{4 \text{ cups}}{1 \text{ qt}} \times \frac{8 \text{ fl oz}}{1 \text{ cup}} = 608 \text{ fl oz.}$$

Method A - Chain conversion factors

Write all conversions in one line and cancel units.

Method B - Convert in stages

19 quarts = 76 cups, then 76 cups = 608 fluid ounces.

SAT decision. Method A is usually safer for compound units; Method B can be easier to explain when the numbers are simple.

Proportions: three reliable methods

A proportion states that two ratios are equal:

$$\frac{a}{b} = \frac{c}{d}.$$

The most important requirement is that corresponding quantities appear in the same positions.

Method A - Scale factor

$$\frac{7}{9} = \frac{x}{45}.$$

Since $9 \rightarrow 45$ multiplies by 5, $7 \rightarrow x$ also multiplies by 5. Thus $x = 35$.

Method B - Unit rate

If 7 units correspond to 9 units of another quantity, one first-unit corresponds to $9/7$ second-units. Build from there.

Method C - Cross multiplication

$$\frac{7}{9} = \frac{x}{45} \Rightarrow 7(45) = 9x \Rightarrow x = 35.$$

Common trap. Cross multiplication is not magic. It works because multiplying both sides of $a/b = c/d$ by bd gives $ad = bc$.

Direct proportion: $y = kx$

Two quantities are directly proportional when their ratio is constant:

$$\frac{y}{x} = k.$$

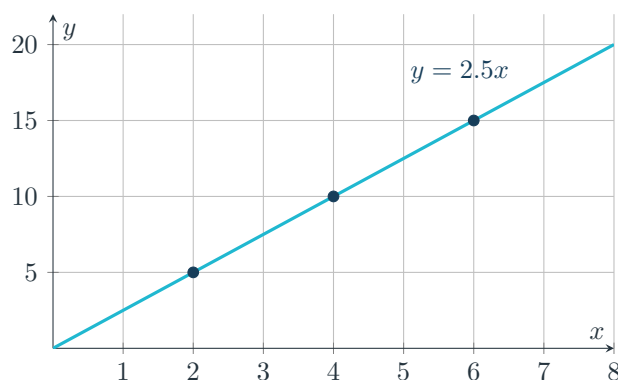
Then

$$y = kx.$$

The constant k is the **constant of proportionality**.

What the graph tells you

A proportional relationship graphs as a straight line through the origin. If the graph is straight but does not pass through $(0, 0)$, it is linear but not proportional.



Find k from a table

If $(x, y) = (3, 12)$, then

$$k = \frac{y}{x} = \frac{12}{3} = 4,$$

so every row must satisfy $y = 4x$.

Tables: how to recognize proportionality

For a proportional table, y/x is constant for every nonzero x .

x	2	5	8	12
y	7	17.5	28	42

Each ratio equals 3.5, so $y = 3.5x$.

Method A - Ratio test

Calculate y/x for two or three rows. If the ratio changes, the table is not proportional.

Method B - Scale test

If x doubles, y must double. If x triples, y must triple.

Method C - Graph test

Plot the points. A proportional relationship lies on a straight line through the origin.

Percent means “per 100”

A percentage is a ratio with denominator 100:

$$27\% = \frac{27}{100} = 0.27.$$

This gives three interchangeable forms:

$$27\% = 0.27 = \frac{27}{100}.$$



The base matters

“30 is 20% of what?” and “What is 20% of 30?” are completely different questions. The word *of* usually points to the whole/base that is multiplied by the percentage.

The three basic percentage problems

All basic percentage questions are versions of

$$\text{part} = \text{percent} \times \text{whole},$$

where the percent is written as a decimal multiplier.

Type A - Find the part

What is 18% of 250?

$$0.18(250) = 45.$$

Type B - Find the whole

45 is 18% of what number?

$$45 = 0.18W \Rightarrow W = 250.$$

Type C - Find the percent

45 is what percent of 250?

$$\frac{45}{250} = 0.18 = 18\%.$$

A useful language map

“is” → part/result, “of” → multiply/base, “what percent” → divide part by whole and multiply by 100.

Percent multipliers: the fastest SAT language

If a quantity increases by $r\%$, multiply by

$$1 + \frac{r}{100}.$$

If it decreases by $r\%$, multiply by

$$1 - \frac{r}{100}.$$



Increase and decrease

Increase 640 by 12%:

$$640(1.12) = 716.8.$$

Decrease 640 by 12%:

$$640(0.88) = 563.2.$$

SAT decision. Multiplier thinking is often faster and safer than “find the change, then add/subtract it.”

Percent increase and percent decrease

Percent change compares the change to the *original* value:

$$\text{percent change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100\%.$$

For a decrease, use the magnitude of the change:

$$\frac{\text{old} - \text{new}}{\text{old}} \times 100\%.$$

From 80 to 104

The change is 24. The original is 80:

$$\frac{24}{80} = 0.30 = 30\%.$$

The denominator is not the new value

Going from 80 to 104 is a 30% increase. Going from 104 back to 80 is

$$\frac{24}{104} \approx 23.1\%$$

decrease. Percent changes are not automatically symmetric.

Reverse percentages

A reverse percentage question gives the final value and asks for the original.

After a 15% increase, the value is 276

A 15% increase means multiplier 1.15:

$$1.15Q = 276 \Rightarrow Q = \frac{276}{1.15} = 240.$$

Method A - Divide by the multiplierFinal = original $\times$ multiplier, so

$$\text{original} = \frac{\text{final}}{\text{multiplier}}.$$

Method B - Percentage equation

If 85% of a number is 255,

$$0.85x = 255 \Rightarrow x = 300.$$

Common trap. Do not simply subtract 15% of the final value. The 15% was measured from the original value, not the final value.

Repeated percentage change

Repeated percentage changes multiply, not add.

Two consecutive increases

A population increases by 10% and then by 20%:

$$Q \rightarrow 1.10Q \rightarrow 1.20(1.10Q) = 1.32Q.$$

The overall increase is 32%, not 30%.

$$\boxed{Q} \quad \times 1.10 \quad \boxed{1.10Q} \quad \times 1.20 \quad \boxed{1.32Q}$$

Increase then decrease

A 20% increase followed by a 20% decrease gives

$$1.20(0.80) = 0.96,$$

so the final value is 4% below the original.

Tax, tip, discount, and markup

These are all multiplier problems.

Situation	Multiplier
8% sales tax	1.08
18% tip	1.18
25% discount	0.75
40% markup	1.40
12% depreciation	0.88

Discount then tax

A \$160 item is discounted 25%, then taxed 8%:

$$160(0.75)(1.08) = 129.60.$$

Common trap. A 25% discount and 8% tax do not combine to a 17% discount. They apply to different bases in sequence.

Percentage points vs percent change

If a survey result rises from 40% to 50%, the increase is

$$50\% - 40\% = 10 \text{ percentage points.}$$

But the percent increase relative to 40% is

$$\frac{50 - 40}{40} = 0.25 = 25\%.$$

These are different measures.

Why the SAT cares

Questions involving rates, survey percentages, or market share may quietly test whether you know the difference between an absolute change in percentage points and a relative percent change.

Nested and chained percentages

Hard percentage questions often connect several quantities.

A percentage of a percentage

Suppose A is 240% of B , and B is 75% of C . Then

$$A = 2.40B, \quad B = 0.75C,$$

so

$$A = 2.40(0.75C) = 1.80C.$$

Thus A is 180% of C .

Method A - Translate each sentence separately

Write one equation per relationship. Then substitute.

Method B - Multiply relationship factors

If $A = 2.4B$ and $B = 0.75C$, the chain factor from C to A is

$$2.4 \times 0.75 = 1.8.$$

Density and derived rates

Density is a rate. Population density, mass density, price density, and pressure all have the same structural idea:

$$\text{rate} = \frac{\text{quantity}}{\text{base measure}}.$$

Quantity

Rate / density

×

Base measure

Method A - Find the rate

$$\text{density} = \frac{\text{population}}{\text{area}}.$$

Method B - Find the quantity

$$\text{population} = \text{density} \times \text{area}.$$

Method C - Find the base

$$\text{area} = \frac{\text{population}}{\text{density}}.$$

Compound rates: multiply when units cancel

Suppose wax has density

$$0.60 \frac{\text{oz}}{\text{in}^3}$$

and costs

$$9 \frac{\$}{\text{oz}}.$$

Then cost per cubic inch is

$$0.60 \frac{\text{oz}}{\text{in}^3} \times 9 \frac{\$}{\text{oz}} = 5.40 \frac{\$}{\text{in}^3}.$$

The ounces cancel. This tells you multiplication is the correct operation.

Unit logic beats memorization

If you are unsure whether to multiply or divide two rates, write the units as fractions and see which operation produces the requested units.

Scale drawings and linear scale factors

A scale statement such as

$$1 \text{ cm on drawing} = 4 \text{ m in reality}$$

creates a proportional relationship.

Map distance

If two points are 7.5 cm apart on the drawing,

$$7.5(4) = 30 \text{ m}.$$

If a drawing is enlarged by 60%, every printed length is multiplied by 1.60. Because the same real-world distance is now spread over more printed inches, the real distance represented by 1 printed inch is divided by 1.60.

Enlarging the paper changes the scale

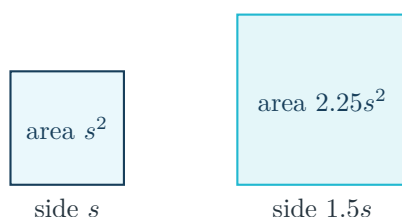
Old scale: 1 inch represents 18 miles. If every drawing length becomes 1.5 times as large, then 1.5 new inches represents the same 18 miles. Therefore 1 new inch represents

$$18 \div 1.5 = 12 \text{ miles.}$$

Area and volume scale factors

If every length is multiplied by k :

$$\text{length factor} = k, \quad \text{area factor} = k^2, \quad \text{volume factor} = k^3.$$



Common trap. A 50% increase in side length does not mean a 50% increase in area. The area multiplier is $1.5^2 = 2.25$, a 125% increase.

Sample proportions and population estimates

If a random sample is representative, the sample proportion can be used to estimate a population count.

Estimate from a sample

In a random sample of 400 customers, 52 prefer a new plan. The sample proportion is

$$\frac{52}{400} = 0.13.$$

For a population of 18,000 customers, the estimate is

$$0.13(18,000) = 2340.$$

Method A - Proportion

$$\frac{52}{400} = \frac{x}{18,000}.$$

Method B - Percentage

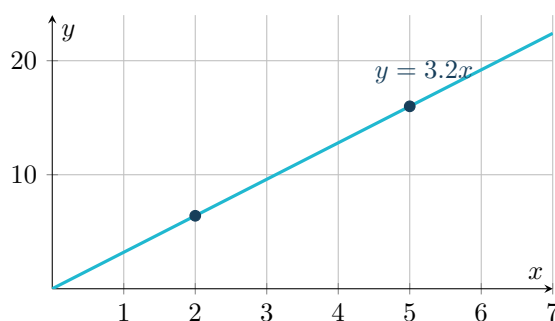
$52/400 = 13\%$, so find 13% of 18,000.

Desmos strategy for ratios and percentages

Most ratio and percentage questions do not need Desmos. Use it when it reduces arithmetic or helps you see a proportional graph.

Desmos Route A - Proportional graph

For $y = 3.2x$, graph the line and inspect points. Every point on the line satisfies $y/x = 3.2$.

**Desmos Route B - Percentage equation**

If a quantity after a 17% increase is 514.8, enter

$$1.17x = 514.8$$

and solve numerically. Algebraically, $x = 514.8/1.17 = 440$.

Desmos Route C - Table check

For a table question, enter the (x, y) pairs. If y/x is constant, the points lie on a line through the origin. Desmos is a visual check, not a replacement for understanding the ratio.

Advanced ratio structures: when two ratios share a quantity

Harder SAT ratio questions sometimes give two relationships that share a middle quantity. The safest strategy is to make the shared quantity match before combining the ratios.

Combine $A : B$ and $B : C$

Suppose

$$A : B = 3 : 4, \quad B : C = 6 : 5.$$

The B -parts are 4 and 6. Their least common multiple is 12.

$$A : B = 9 : 12, \quad B : C = 12 : 10.$$

Therefore

$$A : B : C = 9 : 12 : 10.$$

Method A - Match the shared term

Scale both ratios until the common quantity has the same number of parts. Then join the ratios.

Method B - Use variables

Write $A = \frac{3}{4}B$ and $B = \frac{6}{5}C$. Then

$$A = \frac{3}{4} \cdot \frac{6}{5}C = \frac{9}{10}C,$$

so $A : C = 9 : 10$.

Do not concatenate ratios automatically

From $A : B = 3 : 4$ and $B : C = 6 : 5$, writing $A : B : C = 3 : 4 : 5$ is wrong because the two descriptions of B do not yet match.

Rates in context: cost, productivity, and average rate

The units often reveal the equation before the numbers do.

Price per unit

A 12-ounce package costs \$4.68. The unit price is

$$\frac{4.68}{12} = \boxed{\$0.39 \text{ per ounce}}.$$

If another package costs \$5.60 for 16 ounces, its unit price is \$0.35 per ounce, so it is cheaper per ounce.

Production rate

A machine produces 315 parts in 4.5 hours. Its average production rate is

$$\frac{315}{4.5} = \boxed{70 \text{ parts/hour}}.$$

At that rate, 8 hours gives $70(8) = 560$ parts.

Average rate is total output divided by total input

Do not automatically average two rates arithmetically. If a trip has different speeds for different lengths of time, use

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}.$$

The simple average of the two speeds works only in special situations.

Squared and cubed units: conversion factors must also be squared or cubed

This is a high-value advanced conversion idea. If

$$1 \text{ ft} = 12 \text{ in},$$

then

$$1 \text{ ft}^2 = (12 \text{ in})^2 = 144 \text{ in}^2$$

and

$$1 \text{ ft}^3 = (12 \text{ in})^3 = 1728 \text{ in}^3.$$

Square feet to square inches

Convert 8.5 ft^2 to square inches:

$$8.5 \text{ ft}^2 \times \frac{144 \text{ in}^2}{1 \text{ ft}^2} = \boxed{1224 \text{ in}^2}.$$

Method A - Convert the unit relationship first

From $1 \text{ yd} = 3 \text{ ft}$, infer $1 \text{ yd}^2 = 9 \text{ ft}^2$ before multiplying areas.

Method B - Square the entire conversion factor

$$15 \text{ m}^2 \left(\frac{100 \text{ cm}}{1 \text{ m}} \right)^2 = 150,000 \text{ cm}^2.$$

Common trap. Using 12 instead of 12^2 when converting square feet to square inches is one of the most common unit-conversion errors.

Percentage expressions: read the multiplier backwards

The expression multiplying a quantity tells you the percentage change.

What percent decrease does $0.38z$ represent?

The final amount is 38% of the original. Therefore the amount removed is

$$100\% - 38\% = \boxed{62\%}.$$

What percent increase does $1.27x$ represent?

The multiplier contains the original 100% plus an extra 27%, so the increase is $\boxed{27\%}$.

Multiplier dictionary

$$0.84Q \Rightarrow 16\% \text{ decrease}, \quad 1.06Q \Rightarrow 6\% \text{ increase}, \quad 2.35Q \Rightarrow 135\% \text{ increase}.$$

A multiplier greater than 2 means the final quantity is more than double the original.

Currency conversion plus a fee

Some SAT questions combine a proportional conversion with a percentage fee. Keep the steps separate.

Exchange first, fee second

Suppose 1 unit of currency A equals \$0.08, and a traveler converts 9,000 units. The converted amount is

$$9000(0.08) = \$720.$$

If the service charges a 2.5% fee on the converted amount, the fee is

$$0.025(720) = \$18.$$

The traveler receives

$$720 - 18 = \boxed{\$702}.$$

Method B - One combined multiplier

After conversion, keeping 97.5% of the converted amount means

$$9000(0.08)(0.975) = \boxed{702}.$$

Common trap. Check whether the question asks for the fee, the amount before the fee, or the amount received after the fee.

Percent change from tables and graphs

When data are presented visually, extract the two relevant values first. Only then calculate the percent change.

Table reading

A value is 48 in 2024 and 60 in 2025. The increase is 12, based on the original 48:

$$\frac{12}{48} \times 100\% = \boxed{25\%}.$$

Graph reading

A graph shows rainfall falling from 36 inches to 9 inches. The decrease is 27 inches:

$$\frac{27}{36} \times 100\% = \boxed{75\%}.$$

Read coordinates before calculating

Many wrong answers come from using the vertical-axis labels incorrectly, using the later value as the denominator, or finding the numerical change but forgetting to convert it to a percentage.

Reverse scale reasoning: larger picture, smaller real distance per inch

Scale questions become confusing because two kinds of length are moving in opposite directions.

The invariant idea

The real-world distance represented by a particular segment does not change just because the paper is enlarged. If the printed segment becomes longer, each printed inch must represent fewer real miles.

A 40% enlargement

Old scale: 1 inch represents 21 miles. The print is enlarged by 40%, so the same old 1-inch segment becomes 1.4 inches. Therefore the new scale is

$$\frac{21}{1.4} = \boxed{15 \text{ miles per inch}}.$$

SAT decision. If an enlarged map gives you a larger miles-per-inch value than before, stop and reconsider. Enlargement should make each printed inch represent less real distance.

Answer-choice strategy and reasonableness checks

SAT multiple-choice answers are often built from predictable errors. Use the structure of the choices to diagnose the intended trap.

Check 1 - Units

If the answer choices have units, confirm your final units before comparing numbers.

Check 2 - Direction

After a discount, the answer must be below the original. After a tax or markup, it must be above the original.

Check 3 - Approximate first

18% of 500 is about 90. If your exact result is 9 or 900, the decimal placement is probably wrong.

Check 4 - Test a friendly number

For an abstract percentage expression, let the original quantity be 100. Then the multiplier becomes the final numerical amount immediately.

Master trap list and final decision map

If you see...	Think...
“ratio of A to B”	write A/B , not B/A
“per”	division / unit rate
conversion of units	multiply by factors so units cancel
“is what percent of”	part divided by whole
“increased by $r\%$ ”	multiply by $1 + r$
“decreased by $r\%$ ”	multiply by $1 - r$
final value given	reverse percentage: divide by multiplier
repeated changes	multiply multipliers
density / price per unit	use units to decide multiply or divide
scale drawing enlarged	printed length factor changes inverse real-units-per-inch scale
proportional table	check y/x constant
sample estimate	sample proportion $\times$ population

The five highest-value habits

1. Write units before arithmetic.

2. Identify the original/base in every percent-change problem.

3. Use multipliers for increases and decreases.

4. Keep ratio order consistent.

5. Before cross multiplying, ask whether a scale factor or unit rate is faster.

Exam-style workshop: 15 selected EQB-inspired paraphrases

How these examples are used

The following questions are CF Maths paraphrases/reconstructions inspired by the supplied Educator Question Bank export. Values, wording, and contexts are changed. Each example focuses on a distinct SAT decision rather than reproducing official wording.

Exam Example 1 - Convert a speed

A research drone travels at 3.8 miles per second. What is its speed in miles per hour? *EQB source pattern: d28c29e1. Official wording is not reproduced.* **Method A - unit cancellation.**

$$3.8 \frac{\text{mi}}{\text{s}} \times 60 \frac{\text{s}}{\text{min}} \times 60 \frac{\text{min}}{\text{h}} = 3.8(3600) = \boxed{13,680 \text{ mph}}.$$

Method B - reason in stages. $3.8 \times 60 = 228$ miles/minute, then $228 \times 60 = 13,680$ miles/hour. **Common trap.** Stopping after one factor of 60 gives miles per minute, not miles per hour.

Exam Example 2 - Ratio from a rate story

The ratio of distance d miles to time t seconds is $1 : 6$. If $t = 42$, find d . *EQB source pattern: 3f5398a6. Official wording is not reproduced.*

$$\frac{d}{42} = \frac{1}{6} \Rightarrow d = 7.$$

Quick route. Since $42 = 6(7)$, scale both ratio parts by 7.

Exam Example 3 - Change in density becomes change in population

A region has area $82,500 \text{ km}^2$. Its population density rises from 3.1 to 3.7 people per km^2 . By how many people does the population rise? *EQB source pattern: b4912cc5. Official wording is not reproduced.*

$$\Delta \text{density} = 0.6 \frac{\text{people}}{\text{km}^2}.$$

Therefore

$$0.6(82,500) = \boxed{49,500}.$$

Alternate route. Compute both populations and subtract; the result is the same.

Exam Example 4 - Recover a whole from the remaining percentage

A committee is 20% parents, 35% teachers, and 25% administrators. The remaining 8 members are students. How many committee members are there? *EQB source pattern: 65c49824. Official wording is not reproduced.* The named groups total 80%, so students are 20%.

$$0.20N = 8 \Rightarrow \boxed{N = 40}.$$

Common trap. The 8 students are not 8% of the committee.

Exam Example 5 - Equivalent ratios

The ratio $96 : m$ is equivalent to $6 : 35$. Find m . *EQB source pattern: 825b7490. Official wording is not reproduced.* Since $6 \rightarrow 96$ is $\times 16$, then

$$35 \rightarrow 35(16) = \boxed{560}.$$

Cross multiplication gives the same result.

Exam Example 6 - Chained unit conversion

How many fluid ounces are in 58 quarts? Use 4 cups = 1 quart and 8 fluid ounces = 1 cup. *EQB source pattern: 3310c2ab. Official wording is not reproduced.*

$$58 \times 4 \times 8 = \boxed{1856 \text{ fl oz}}.$$

Unit route.

$$58 \text{ qt} \times \frac{4 \text{ cups}}{1 \text{ qt}} \times \frac{8 \text{ fl oz}}{1 \text{ cup}}.$$

Exam Example 7 - Reverse percentage

In a sample, 72% of the items are defective. There are 90 defective items. How many items are in the sample? *EQB source pattern: 05195d8a. Official wording is not reproduced.*

$$0.72N = 90 \Rightarrow N = \frac{90}{0.72} = \boxed{125}.$$

Exam Example 8 - Enlarged scale drawing

On a square drawing, 1 inch represents 24 miles. A larger version is printed with every side length 50% longer. On the larger version, how many actual miles does 1 inch represent? *EQB source pattern: 28ff3966. Official wording is not reproduced.* The print scale factor is 1.50. The same 24 miles now occupies 1.5 inches, so

$$\frac{24}{1.5} = \boxed{16 \text{ miles per inch}}.$$

Exam Example 9 - Nested percentages

Positive number A is 180% of B , and B is 70% of C . What percent of C is A ? *EQB source pattern: 4aaa9c42. Official wording is not reproduced.*

$$A = 1.8B, \quad B = 0.70C.$$

Thus

$$A = 1.8(0.70C) = 1.26C,$$

so A is $\boxed{126\%}$ of C .

Exam Example 10 - Percent increase from a table

A wage rises from \$12.50 to \$15.00. What is the percent increase? *EQB source pattern: bd90f87e. Official wording is not reproduced.*

$$\frac{15.00 - 12.50}{12.50} \times 100\% = \frac{2.50}{12.50} \times 100\% = \boxed{20\%}.$$

Exam Example 11 - A second percent increase is based on the new value

Subscriptions rise from 8,000 to 8,400 in one year. The next year's percent increase is half as large. Estimate the next year's subscriptions. *EQB source pattern: 0ea56bb2. Official wording is not reproduced.* First increase:

$$\frac{8400 - 8000}{8000} = 0.05 = 5\%.$$

Next increase is 2.5%:

$$8400(1.025) = \boxed{8610}.$$

Common trap. Do not add half of the previous numerical increase; the question halves the percentage rate.

Exam Example 12 - Multiply compound rates

A material has density 0.72 ounce per cubic inch and costs \$7.50 per ounce. Find the cost per cubic inch. *EQB source pattern: 8e528129. Official wording is not reproduced.*

$$0.72 \frac{\text{oz}}{\text{in}^3} \times 7.50 \frac{\$}{\text{oz}} = \boxed{5.40 \frac{\$}{\text{in}^3}}.$$

The ounces cancel, confirming multiplication.

Exam Example 13 - Area from population density

A town has population 76,500 and population density 255 people per square mile. Find its area. *EQB source pattern: 15617f62. Official wording is not reproduced.*

$$255 = \frac{76,500}{A} \Rightarrow A = \frac{76,500}{255} = \boxed{300 \text{ mi}^2}.$$

Exam Example 14 - Chain three percentage relationships

P is 225% of Q . Q is 64% of R . If R is $x\%$ of P , find x . *EQB source pattern: 993000da. Official wording is not reproduced.*

$$P = 2.25Q = 2.25(0.64R) = 1.44R.$$

So

$$R = \frac{1}{1.44}P \approx 0.69444P.$$

Therefore R is approximately $\boxed{69.4\%}$ of P .

Exam Example 15 - Sales tax multiplier

A jacket costs \$84 before 7.5% sales tax. What is the total price? *EQB source pattern: 8705ecba. Official wording is not reproduced.*

$$84(1.075) = \boxed{\$90.30}.$$

Alternate route. Tax = $84(0.075) = 6.30$, then add to 84.

Final chapter checklist

Before leaving Chapter 2, make sure you can answer all of these without guessing:

- Can I keep ratio order correct?
- Can I move between ratio, fraction, decimal, percentage, and multiplier forms?
- Can I convert compound units by cancellation?
- Can I recognize $y = kx$ from a graph or table?
- Can I identify the original value in a percent-change problem?
- Can I reverse a percentage by dividing by the multiplier?
- Can I combine repeated changes by multiplying multipliers?
- Can I use units to decide whether to multiply or divide density/rate quantities?
- Can I distinguish length, area, and volume scale factors?
- Can I estimate a population using a representative sample proportion?

Next step

Use the separate worksheet without looking at the solutions. Mark every question where the setup - not the arithmetic - caused difficulty. Those setup errors tell you which theory section to revisit.

Additional SAT-Style Multiple Choice

Chapter 2: Ratios, Percentages and Proportions

These are new CF Maths questions built from common Digital SAT patterns involving ratios, rates, conversions, proportions, percentages, scale factors, density, and multi-step change. Focus on the base quantity and units before calculating.

- The ratio of red to blue marbles is 3 : 5. If there are 72 marbles in total, how many are red?
 - 24
 - 27
 - 40
 - 45
- A machine produces 126 parts in 3.5 hours. At this constant rate, how many parts does it produce per hour?
 - 32
 - 36
 - 40
 - 44
- A cyclist travels at 18 miles per hour. What is this speed in miles per minute?
 - 0.2
 - 0.3
 - 1.08
 - 30
- A liquid has density 1.25 grams per milliliter. What is the mass of 80 milliliters?
 - 64 g
 - 81.25 g
 - 100 g
 - 125 g
- If $\frac{x}{18} = \frac{14}{9}$, what is x ?
 - 14
 - 18
 - 28
 - 36
- y is directly proportional to x . If $y = 42$ when $x = 6$, what is y when $x = 11$?
 - 66
 - 70
 - 72
 - 77
- What is 35% of 240?
 - 72
 - 84

- C. 96
D. 105
8. If 72 is 60% of a number, what is the number?
- A. 108
B. 120
C. 132
D. 144
9. A price rises from \$80 to \$92. What is the percent increase?
- A. 12%
B. 15%
C. 18%
D. 20%
10. A quantity decreases from 250 to 190. What is the percent decrease?
- A. 20%
B. 24%
C. 30%
D. 60%
11. After a 20% discount, a jacket costs \$96. What was the original price?
- A. \$115.20
B. \$120
C. \$124
D. \$128
12. A population increases by 10% and then decreases by 10%. Compared with the original population, the final population is
- A. 1% lower
B. the same
C. 1% higher
D. 2% lower
13. A \$50 item is discounted by 20% and then a 7.5% sales tax is applied to the discounted price. What is the final price?
- A. \$40.00
B. \$42.50
C. \$43.00
D. \$46.50
14. A store marks an item up by 30% from a wholesale price of \$70. What is the selling price?
- A. \$79
B. \$91
C. \$100
D. \$130

15. In a random sample of 250 voters, 145 support a proposal. Based on this sample, about how many of 8,000 voters would be expected to support it?
- A. 3,200
 - B. 4,000
 - C. 4,640
 - D. 5,800
16. A town has 54,600 residents in an area of 140 square miles. What is the population density?
- A. 260
 - B. 350
 - C. 390
 - D. 490
- people per square mile
17. On a map, 2.5 cm represents 18 km. How many kilometers are represented by 7.5 cm?
- A. 36
 - B. 45
 - C. 54
 - D. 60
18. A square is enlarged so each side is multiplied by 1.4. By what factor is its area multiplied?
- A. 1.4
 - B. 1.8
 - C. 1.96
 - D. 2.8
19. A cube's side length is multiplied by 1.5. By what factor is its volume multiplied?
- A. 2.25
 - B. 3.00
 - C. 3.375
 - D. 4.50
20. One currency unit is worth \$0.80. A bank charges a 2% fee on the dollar value of the exchange. What is the total dollar cost to buy 500 currency units?
- A. \$400
 - B. \$404
 - C. \$408
 - D. \$410
21. A club's membership is 25% students. Of the students, 40% are first-year students. What percent of the entire club are first-year students?
- A. 10%
 - B. 15%
 - C. 25%
 - D. 40%
22. The ratio of boys to girls in a group is 4 : 7. After 6 boys join, the ratio becomes 5 : 7. How many girls are in the group?

- A. 28
- B. 35
- C. 42
- D. 49

23. The table follows a proportional relationship: $(x, y) = (3, 18), (5, 30), (12, k)$. What is k ?

- A. 60
- B. 66
- C. 72
- D. 90

24. A car travels 150 miles at 50 mph and then 120 miles at 60 mph. How long does the entire trip take?

- A. 4 h
- B. 4.5 h
- C. 5 h
- D. 5.5 h

Answer Key

For every missed question, write the base quantity and the units before reviewing the arithmetic. Most errors in this chapter come from choosing the wrong denominator or applying a percent to the wrong starting value.

- | | | | |
|------|-------|-------|-------|
| 1. B | 7. B | 13. C | 19. C |
| 2. B | 8. B | 14. B | 20. C |
| 3. B | 9. B | 15. C | 21. A |
| 4. C | 10. B | 16. C | 22. C |
| 5. C | 11. B | 17. C | 23. C |
| 6. D | 12. A | 18. C | 24. C |