

CF MATHS

SAT Math Lecture Notes

Functions

Read inputs, outputs, graphs, tables, linear models, quadratics, and exponential models as different views of the same idea

■ Expanded theory edition

- complete function notation, graph, table, and model interpretation
- Method A / Method B / Method C decision-making throughout
- linear functions in depth, then nonlinear functions as graph families
- quadratic and exponential features without unnecessary algebra overload
- Desmos-style visual routes and transformation sketches
- 15 fully worked exam-style reconstructions plus 20-question MC review
- separate 35-question worksheet with solutions collected only at the end

Built from the supplied SAT Educator Question Bank function pool and common SAT workbook patterns. Official question wording is not reproduced. All practice wording, values, and contexts in this edition are newly written for CF Maths.

Chapter roadmap

The big story

A function is a rule that turns an input into exactly one output. The SAT may hide that rule in an equation, a graph, a table, or a story. Your job is to recognize that these are not four different topics. They are four ways of describing the same input-output relationship.

A strong student keeps asking two questions: *What does the input mean?* *What does the output mean?* Once those are clear, notation and graphs become much easier.

1. What a function really is
2. Function-machine thinking
3. Evaluating $f(a)$
4. Finding an input from an output
5. Tables and constant functions
6. Graphs and intercepts
7. Domain in context
8. Linear functions $mx + b$
9. Slope as rate of change
10. Intercept as initial value
11. Building lines from points/tables
12. Comparing representations
13. Linear vs proportional
14. Recognizing nonlinear functions
15. Quadratic features
16. Exponential features
17. Function transformations
18. Desmos strategy
19. Master decision map
20. 15 worked exam-style examples
21. 20-question MC review

What is a function?

Imagine a machine. You put in a number, the machine follows a rule, and one number comes out.

$$\boxed{x} \longrightarrow \boxed{\text{rule}} \longrightarrow \boxed{f(x)}$$

If the rule is $f(x) = 3x + 5$, then an input of 4 produces

$$f(4) = 3(4) + 5 = 17.$$

The most important sentence

$f(4) = 17$ means: *when the input is 4, the output is 17*. It does not mean $f \times 4$. The parentheses are part of function notation.



Method A - Substitute directly

For $f(x) = 5x - 7$, find $f(6)$:

$$f(6) = 5(6) - 7 = 23.$$

This is the fastest route when the formula is already given.

Method B - Treat the input as a complete object

If $g(x) = 2x^2 + 1$, then

$$g(a + 1) = 2(a + 1)^2 + 1.$$

Do not replace only one appearance of x . Every x in the rule receives the new input.

Trap: output vs input

If $f(8) = 31$, then 8 is the input and 31 is the output. SAT questions often reverse the direction and ask, "For what value of x is $f(x) = 31$?" That is now an equation-solving problem.

Finding the input from the output

Suppose

$$f(x) = 7x + 4$$

and the question asks for the input that gives output 53. Write

$$7x + 4 = 53,$$

so $x = 7$.

Method A - Set the function equal to the target

"For what x is $f(x) = T$?" means replace $f(x)$ by its formula and solve

$$\text{formula} = T.$$

Method B - Graph the target line

In Desmos, enter $y = f(x)$ and $y = T$. The x -coordinate of the intersection is the required input. This is especially useful when f is nonlinear or messy.

Tables: functions without formulas

A table can define a function just as completely as an equation.

x	0	1	2	3
$f(x)$	5	8	11	14

The output increases by 3 whenever the input increases by 1. So the function is linear with slope 3. Since $f(0) = 5$,

$$f(x) = 3x + 5.$$

A useful table question

For a linear function, the first differences in the output are constant when the input changes by equal amounts. For an exponential function, the output often changes by a constant *factor* instead.

Method A - Difference test for linearity

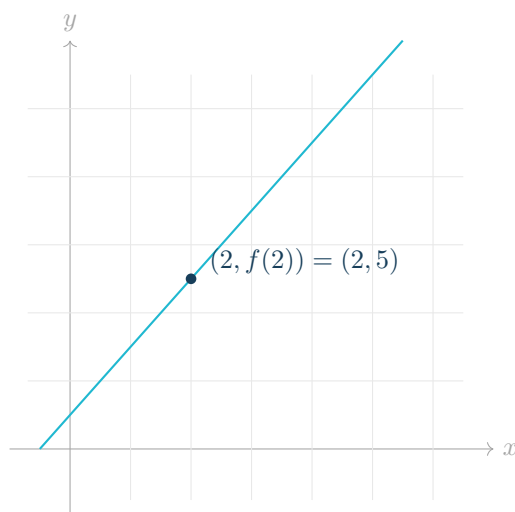
Equal input steps $\Rightarrow$ check output differences. Constant differences suggest a linear model.

Method B - Ratio test for exponential behavior

Equal input steps $\Rightarrow$ check output ratios. A constant ratio suggests an exponential model.

Graphs: read before you calculate

A graph of $y = f(x)$ is a picture of all input-output pairs $(x, f(x))$.



Intercepts

The y -intercept occurs when $x = 0$. It is the output before any input has accumulated. The x -intercept occurs when the output is 0.

$$y\text{-intercept: } (0, f(0)), \quad x\text{-intercept: } f(x) = 0.$$

Context matters

An intercept can have a clean algebraic meaning but an impossible real-world meaning. A model for height after launch may not be meaningful for negative time. Always respect the stated domain.

Domain: which inputs are allowed?

The **domain** is the set of possible inputs. SAT questions often define domain through context rather than formal notation.

If $C(n) = 25 + 8n$ is the cost of buying n tickets, then n is usually a nonnegative whole number. Values such as $n = 2.7$ are algebraically possible but contextually meaningless.

Read the units

If the input is number of objects, expect whole numbers. If the input is time, distance, temperature, or money, non-integer values may be meaningful. Units often reveal the intended domain faster than algebra does.

Linear functions: constant additive change

A linear function has the form

$$f(x) = mx + b.$$

Here m is the constant rate of change and $b = f(0)$ is the initial value.

- m change in output for each increase of 1 in input; units are output-units per input-unit
- b output when input is 0; often a starting amount, fixed fee, initial height, or baseline

Method A - Equation already given

For $P(t) = 1200 + 85t$, the initial value is 1200 and the rate is 85 per unit of t .

Method B - Two points

For points (x_1, y_1) and (x_2, y_2) ,

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Then use $y = mx + b$ with either point to find b .

Method C - Table shortcut

If the table uses equal input steps, slope is

$$\frac{\text{change in output}}{\text{change in input}}.$$

Then use a row to recover the intercept if needed.

Slope is a rate, not just a number

Suppose $D(p) = 20000 - 250p$, where p is a price increase measured in dollars and D is demand in units. The slope -250 means demand decreases by 250 units for each \$1 increase in price.

Units expose the meaning

A slope always has units

$$\frac{\text{output units}}{\text{input units}}.$$

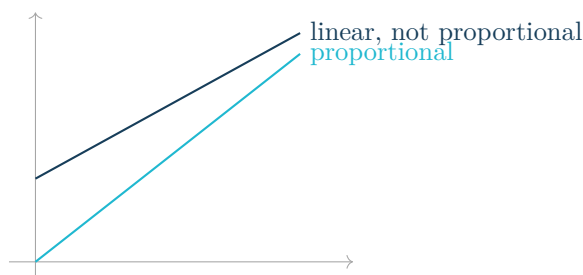
If your interpretation has no units or the wrong units, it is probably wrong.

Linear but not proportional

A proportional relationship has the special form

$$y = kx,$$

so it passes through the origin. A general linear relationship $y = mx + b$ with $b \neq 0$ is linear but not directly proportional.



Building a linear model from a story

Many SAT models are just

$$\text{output} = \text{starting amount} + (\text{rate})(\text{input}).$$

Fixed fee plus usage

A bike rental costs \$18 to start and \$7 per hour. If h is hours,

$$C(h) = 18 + 7h.$$

The 18 is a fixed fee; the 7 is dollars per hour.

Method A - Identify the starting amount first

Look for words such as initial, starting, base fee, when $x = 0$, or already in the account. This becomes the constant term.

Method B - Identify the repeated change

Look for per, each, every, for each additional, increases by, decreases by. This becomes the slope.

Comparing representations

The SAT may give one function as a graph and another as a table. Do not convert everything unless necessary. Extract only the requested feature.

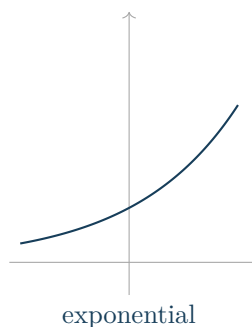
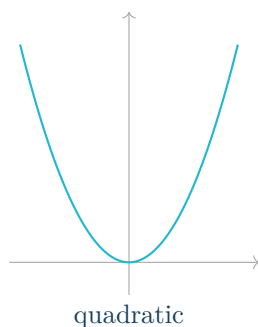
Feature-first thinking

If the question asks which function has the greater rate of change, find only the slopes. If it asks which has the larger initial value, find only $f(0)$. If it asks where they are equal, then solve or graph both.

$$\begin{array}{c|ccc} x & 0 & 2 & 4 \\ \hline g(x) & 9 & 15 & 21 \end{array} \Rightarrow m_g = \frac{15-9}{2} = 3.$$

Nonlinear functions: the rate changes

A nonlinear function does not have one constant slope everywhere. The two most important SAT families are quadratics and exponentials.



Quadratic functions: read the form

A quadratic has a parabola-shaped graph. Three forms reveal different information.

$$\text{standard: } f(x) = ax^2 + bx + c \quad f(0) = c$$

$$\text{factored: } f(x) = a(x - r_1)(x - r_2) \quad x\text{-intercepts } r_1, r_2$$

$$\text{vertex: } f(x) = a(x - h)^2 + k \quad \text{vertex } (h, k)$$

Method A - Read the feature directly

Do not expand vertex form if the question asks for the vertex. Do not expand factored form if the question asks for zeros.

Method B - Use symmetry

If the zeros are r_1 and r_2 , the axis of symmetry is

$$x = \frac{r_1 + r_2}{2}.$$

That x -value is also the input where a parabola reaches its maximum or minimum.

Maximum or minimum?

If $a > 0$, the parabola opens upward and the vertex is a minimum. If $a < 0$, it opens downward and the vertex is a maximum.

Exponential functions: constant multiplicative change

An exponential model often has the form

$$f(t) = A(b)^t.$$

Here $A = f(0)$ is the initial amount and b is the growth/decay factor.

$b > 1$ growth; e.g. $b = 1.06$ means 6% growth per step

$0 < b < 1$ decay; e.g. $b = 0.92$ means 8% decay per step

A initial output at input 0

Method A - Translate percent to factor

Increase by $r\%$: multiply by $1 + r/100$. Decrease by $r\%$: multiply by $1 - r/100$.

Method B - Read the time unit carefully

If the exponent is $t/3$, the multiplicative change occurs every 3 time units. If the input itself is measured in hundreds of meters, one input unit means 100 meters.

Linear versus exponential: amount vs factor

Suppose both start at 100.

$$L(t) = 100 + 20t, \quad E(t) = 100(1.20)^t.$$

The linear model adds 20 each step. The exponential model multiplies by 1.20 each step.

t	0	1	2	3
$L(t)$	100	120	140	160
$E(t)$	100	120	144	172.8

Same first step does not mean same model

A linear and exponential model can agree at two inputs and then separate. Determine whether the change is additive or multiplicative.

Transformations: move the graph without rebuilding it

If $y = f(x)$ is known, common transformations are:

new function	effect
$f(x) + k$	up k
$f(x) - k$	down k
$f(x - h)$	right h
$f(x + h)$	left h
$af(x)$	vertical scaling by $ a $; reflect if $a < 0$

The horizontal sign feels backwards

$f(x - 3)$ moves the graph right 3 because the same original input is reached when the new x is 3 larger. Think “what new input makes the inside equal the old input?”

Desmos strategy for functions

Route A - Evaluate visually

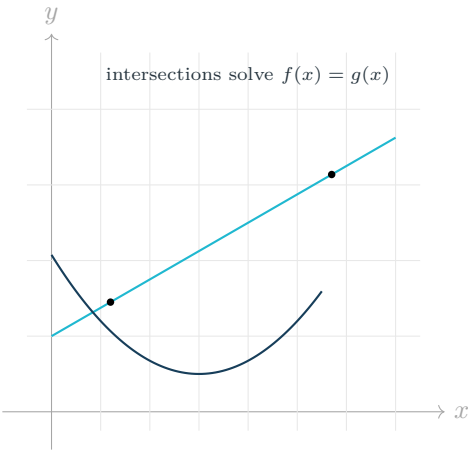
Enter $y = f(x)$. To estimate $f(4)$, either substitute in an expression line or inspect the graph at $x = 4$. For exact arithmetic, direct substitution is usually faster.

Route B - Solve $f(x) = k$

Graph $y = f(x)$ and $y = k$. Each intersection is an input that produces output k . For a quadratic there may be two such inputs.

Route C - Compare two functions

Graph $y = f(x)$ and $y = g(x)$. Their intersections solve $f(x) = g(x)$. Use this when the expressions are awkward or the question asks approximately where models agree.



Master decision map

What is the question actually asking?

1. **Evaluate a function?** Substitute the input.
2. **Find the input for a given output?** Set the formula equal to the target.
3. **Interpret a coefficient?** Attach units; slope is output per input.
4. **Interpret a constant?** Check the output when input is 0.
5. **Build a linear model?** Starting amount + rate $\times$ input.
6. **Table is linear?** Constant first differences.
7. **Table is exponential?** Constant multiplicative factor.
8. **Quadratic feature?** Use the form that displays it directly.
9. **Compare functions?** Extract only the requested feature or graph both.
10. **Context?** Respect units and domain.

Ten traps to avoid

- | | |
|---|---|
| 1. reading $f(x)$ as multiplication | 6. interpreting x outside the context domain |
| 2. swapping input and output | 7. expanding a quadratic unnecessarily |
| 3. using slope as an initial value | 8. treating exponential growth as additive |
| 4. assuming every line is proportional | 9. reading $f(x - h)$ as a left shift |
| 5. forgetting that $f(0)$ is the y -intercept | 10. using Desmos before translating the story correctly |

15 exam-style worked examples

Worked Example 1 - Evaluate a linear function

Let $f(x) = 24x + 17$. Find $f(3)$.

$$f(3) = 24(3) + 17 = 89.$$

SAT decision. Substitute before doing anything else. *Question-bank pattern reference: linear-function evaluation. Wording, values, and context have been rewritten.*

Worked Example 2 - Work backward from the output

Let $g(x) = 9x - 6$. For what value of x is $g(x) = 75$?

$$9x - 6 = 75 \Rightarrow 9x = 81 \Rightarrow \boxed{x = 9}.$$

SAT decision. “For what input?” means solve an equation.

Worked Example 3 - Find a parameter

For $f(x) = 5x + b$, suppose $f(8) = 44$. Then

$$40 + b = 44 \Rightarrow \boxed{b = 4}.$$

SAT decision. A known function value gives one point on the graph.

Worked Example 4 - Table to equation

A linear function satisfies $f(-2) = -7$ and $f(4) = 17$. Its slope is

$$m = \frac{17 - (-7)}{4 - (-2)} = 4.$$

Using $17 = 4(4) + b$ gives $b = 1$, so

$$\boxed{f(x) = 4x + 1}.$$

Worked Example 5 - Context model

A service costs \$120 initially plus \$35 per month. If m is months,

$$\boxed{C(m) = 120 + 35m}.$$

The 120 is the initial fee; 35 is dollars per month.

Worked Example 6 - Intercepts

For $h(x) = 4x + 28$, the y -intercept is $(0, 28)$. For the x -intercept,

$$4x + 28 = 0 \Rightarrow x = -7,$$

so the intercept is $(-7, 0)$.

Worked Example 7 - Compare rates from two representations

Function f has equation $f(x) = 2.5x + 8$. Function g has table $g(1) = 10$ and $g(5) = 24$. Then

$$m_g = \frac{24 - 10}{5 - 1} = 3.5.$$

So g has the greater rate of change.

Worked Example 8 - Quadratic minimum from zeros

Suppose $q(x) = (x - 2)(x - 10)$. The zeros are 2 and 10, so the axis of symmetry is

$$x = \frac{2 + 10}{2} = 6.$$

Because the leading coefficient is positive, q reaches its minimum when $x = 6$.

Worked Example 9 - Vertex form

For $r(x) = -3(x - 4)^2 + 11$, the vertex is $(4, 11)$. Since the coefficient of the squared term is negative, 11 is the maximum output.

Worked Example 10 - Exponential growth

A culture has $B(t) = 3600(1.08)^t$. The initial amount is $B(0) = 3600$. The base 1.08 means 8% growth per time unit.

Worked Example 11 - Exponential decay with a scaled input

Suppose $N(d) = 500(0.90)^d$, where d is depth measured in hundreds of meters. Each additional 100 meters multiplies the output by 0.90, so it decreases by 10% per 100 meters.

Worked Example 12 - Linear or exponential from a table

For outputs 40, 52, 67.6, 87.88 at inputs 0, 1, 2, 3, each output is multiplied by 1.3. Therefore the relationship is exponential, not linear.

Worked Example 13 - Transformation

If $y = f(x)$ is shifted right 5 and up 2, the new function is

$$g(x) = f(x - 5) + 2.$$

Worked Example 14 - Domain in context

A function $T(n) = 15 + 6n$ gives a total cost for n identical items. If at most 12 items may be purchased, the contextual domain is

$$n \in \{0, 1, 2, \dots, 12\}.$$

Worked Example 15 - Read a function statement

If $M(12) = 48$, and $M(t)$ is mass in grams t days after an experiment begins, then the statement means the predicted mass after 12 days is 48 grams. The input and output units are part of the interpretation.

20-question multiple-choice review

These are original CF Maths SAT-style questions. Complete them before looking at the answer key.

1. If $f(x) = 6x - 5$, what is $f(7)$?
 - A. 32
 - B. 37
 - C. 42
 - D. 47
2. If $g(x) = 4x + 9$ and $g(x) = 45$, what is x ?
 - A. 7
 - B. 8
 - C. 9
 - D. 11
3. A linear function satisfies $f(0) = 12$ and has slope -3 . Which equation defines it?
 - A. $f(x) = 12x - 3$
 - B. $f(x) = -3x + 12$
 - C. $f(x) = 3x - 12$
 - D. $f(x) = -12x + 3$
4. A line contains $(2, 9)$ and $(6, 21)$. What is its slope?
 - A. 2
 - B. 3
 - C. 4
 - D. 6
5. Which table is consistent with an exponential relationship?
 - A. $(0, 2), (1, 5), (2, 8)$
 - B. $(0, 3), (1, 6), (2, 12)$
 - C. $(0, 4), (1, 7), (2, 10)$
 - D. $(0, 5), (1, 9), (2, 13)$
6. For $p(x) = 2(x - 3)^2 - 8$, what is the vertex?
 - A. $(-3, -8)$
 - B. $(3, -8)$
 - C. $(-3, 8)$
 - D. $(3, 8)$
7. The zeros of a quadratic are -2 and 8 . What is the axis of symmetry?
 - A. $x = 2$
 - B. $x = 3$
 - C. $x = 5$
 - D. $x = 6$
8. If $A(t) = 900(1.04)^t$, what does 1.04 represent?
 - A. Initial value
 - B. 4% growth per step
 - C. 104% growth per step
 - D. 4-unit additive growth
9. If $Q(t) = 700(0.85)^t$, the quantity decreases by what percent per step?

- A. 15%
 - B. 17.5%
 - C. 85%
 - D. 115%
10. The graph of $y = f(x)$ is shifted left 4. Which expression represents the new graph?
- A. $f(x - 4)$
 - B. $f(x + 4)$
 - C. $f(x) - 4$
 - D. $f(x) + 4$
11. A cost function is $C(h) = 75 + 18h$. What is $C(0)$?
- A. 0
 - B. 18
 - C. 75
 - D. 93
12. For the same function $C(h) = 75 + 18h$, what does 18 represent?
- A. Initial fee
 - B. Cost per hour
 - C. Total after one hour
 - D. Maximum cost
13. A linear function has table values $(1, 8), (3, 14), (5, 20)$. What is $f(0)$?
- A. 3
 - B. 5
 - C. 6
 - D. 7
14. Which function is directly proportional?
- A. $y = 4x$
 - B. $y = 4x + 3$
 - C. $y = x^2$
 - D. $y = 4^x$
15. If $r(x) = -2(x + 1)^2 + 9$, what is the maximum value of r ?
- A. -2
 - B. -1
 - C. 2
 - D. 9
16. A population model is $P(t) = 12000(1.025)^t$. What is $P(0)$?
- A. 1.025
 - B. 120
 - C. 12,000
 - D. 12,300
17. Function f has slope 5. Function g has $g(2) = 9$ and $g(8) = 33$. Which has greater slope?
- A. f
 - B. g
 - C. same slope
 - D. not enough information
18. If $f(3) = 11$, which point lies on the graph of $y = f(x)$?
- A. $(11, 3)$

- B. $(3, 11)$
- C. $(0, 11)$
- D. $(3, 0)$

19. A quadratic opens upward and has vertex $(5, -7)$. Which statement must be true?

- A. Its maximum is -7
- B. Its minimum is -7
- C. Its y -intercept is -7
- D. It has no x -intercepts

20. If $f(x) = 3x + 4$ and $g(x) = x^2$, for which positive x do the graphs intersect?

- A. 1
- B. 2
- C. 3
- D. 4

Multiple-choice answer key

1. B	6. B	11. C	16. C
2. C	7. B	12. B	17. A
3. B	8. B	13. B	18. B
4. B	9. A	14. A	19. B
5. B	10. B	15. D	20. D

Final strategy

Functions become manageable when you stop treating notation, graphs, tables, and stories as separate chapters. They are all different windows onto one relationship. Translate the question into the feature you need, then choose the shortest representation.